From Fermi to non-Fermi liquids

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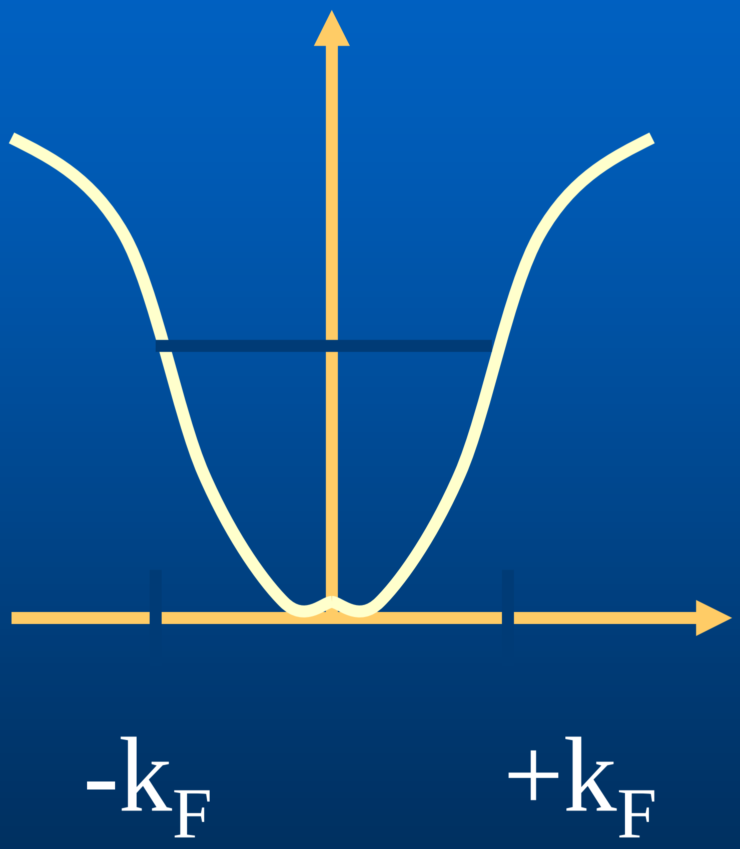
References on Fermi liquids

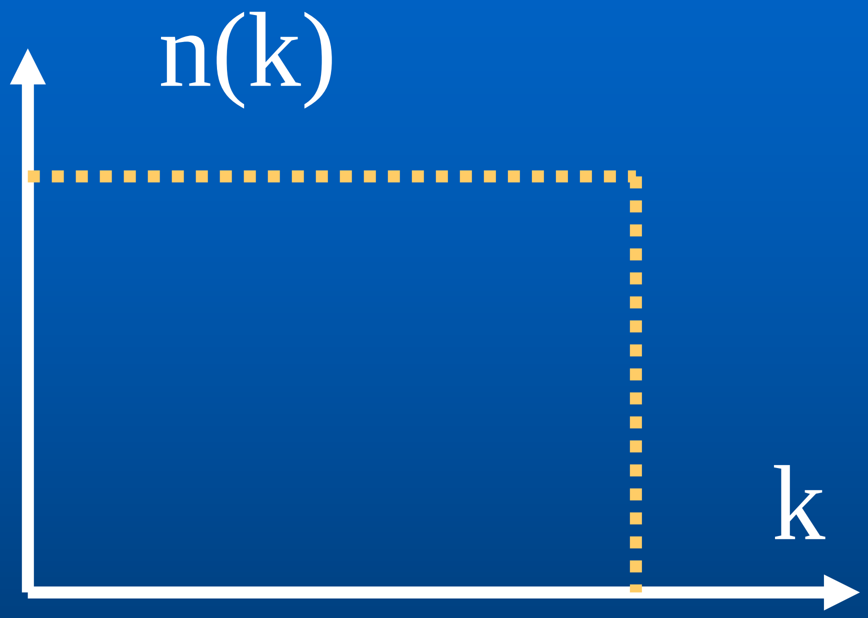
- P. Nozières, ``Theory of interacting Fermi systems'', (1961).
- G.D. Mahan, ``Many particle physics'', Plenum (1981).
- R. Shankar, Rev. Mod. Phys 66, 129 (1994).

Questions

- Understood: free electrons
- Real systems : Coulomb interaction
- Properties of realistic systems
- Free electron theory works quite well

Free electrons : basics





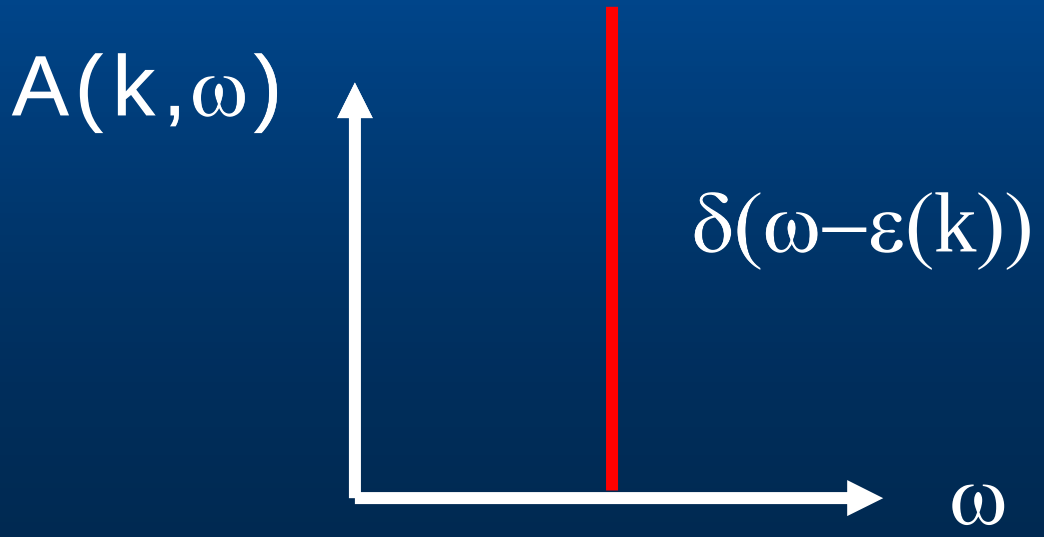
Individual excitations: fermions (particles or holes)

Properties

- Thermodynamics

$$C_V \propto T \qquad\qquad \kappa = Cste$$

- Spectral function



How to probe ?

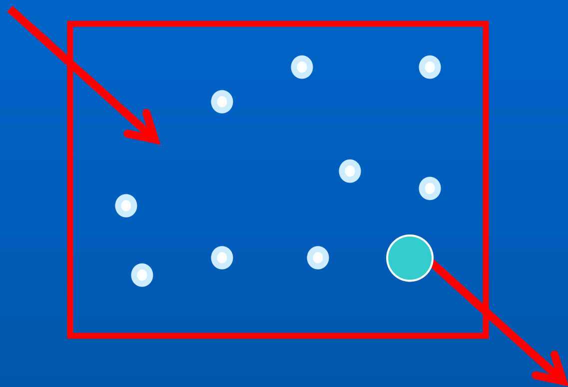
- Specific heat $C_V = \gamma T$
- Responses:

  NMR, Neutrons (spin correlations)

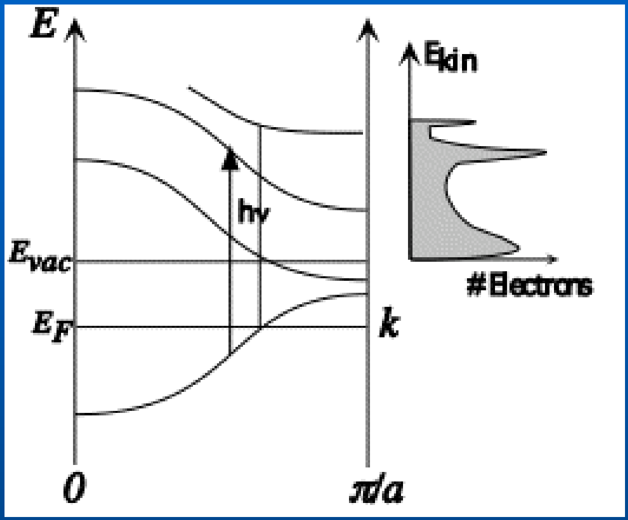
  Transport (charge excitations)

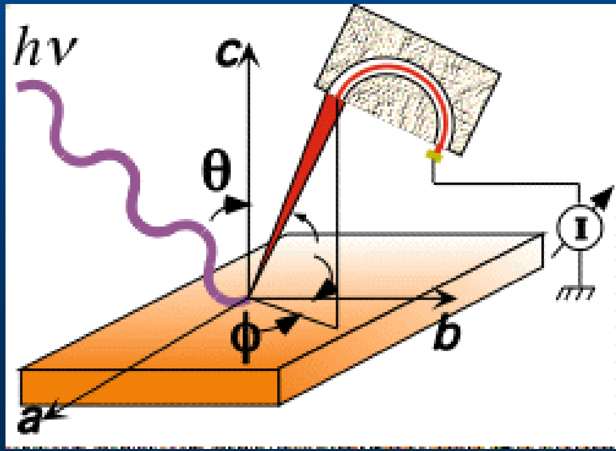
  Photoemission (single particles)

Photoemission



$$\langle \psi(k,\omega)\psi^{\dagger}(k,\omega)\rangle$$



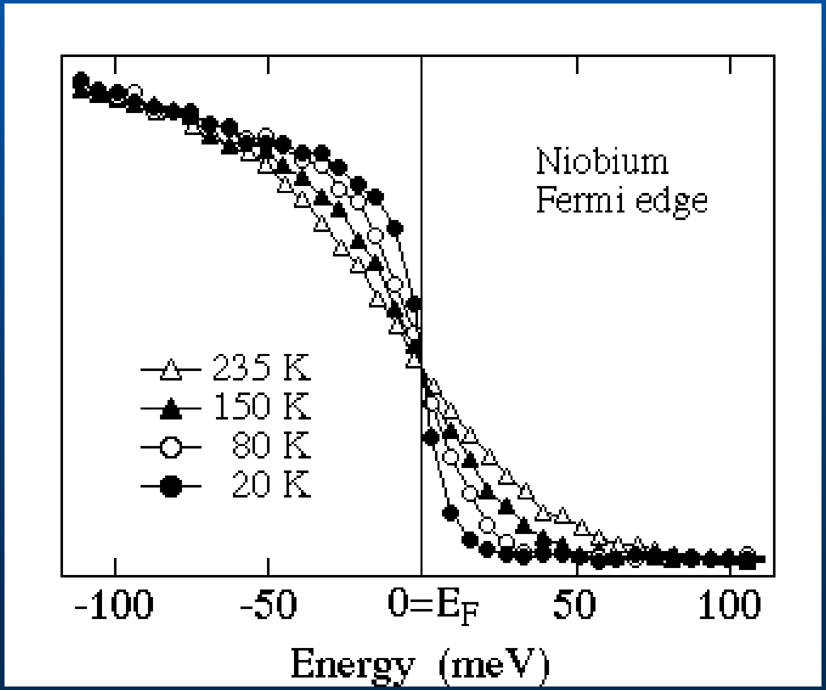


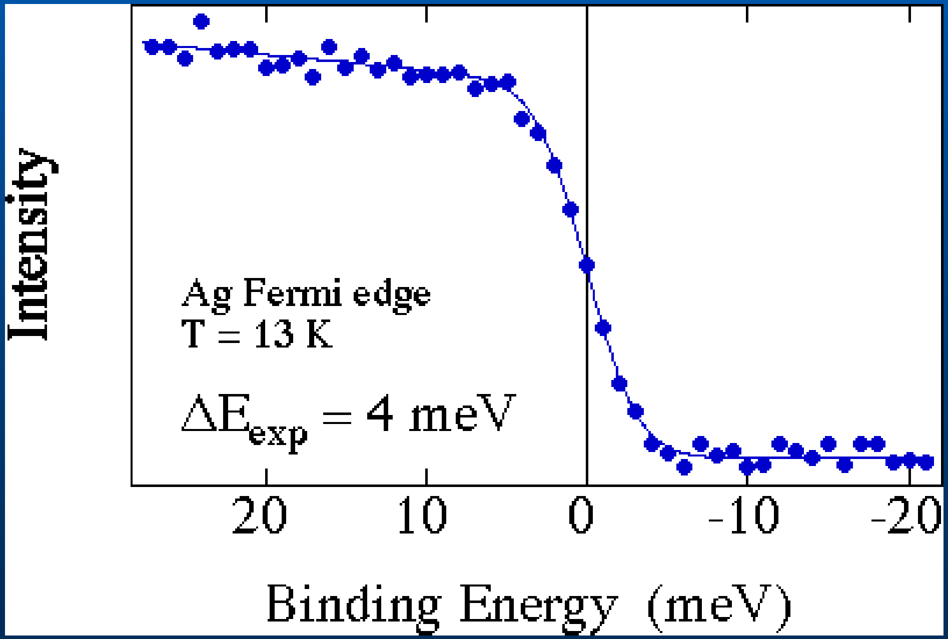
$$k_{//} = \sqrt{\frac{2mE_{kin}}{\hbar}}\sin\theta$$

Measures Spectral function A(k,ω)

``Free electrons'' works

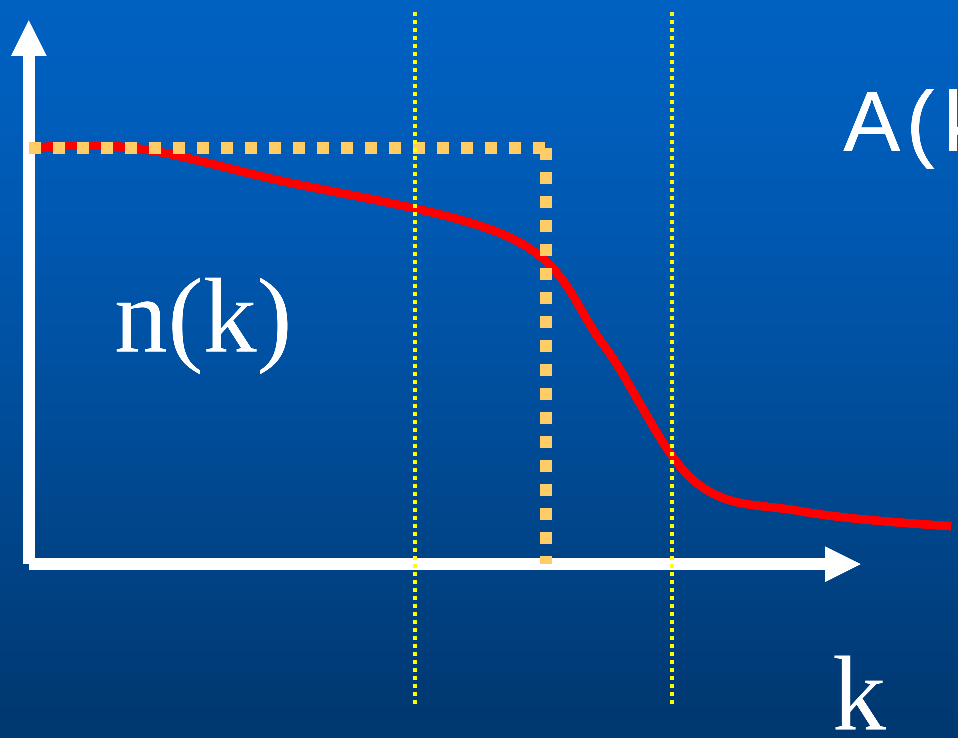
$C_V = \gamma\, T$ κ = cste χ = cste



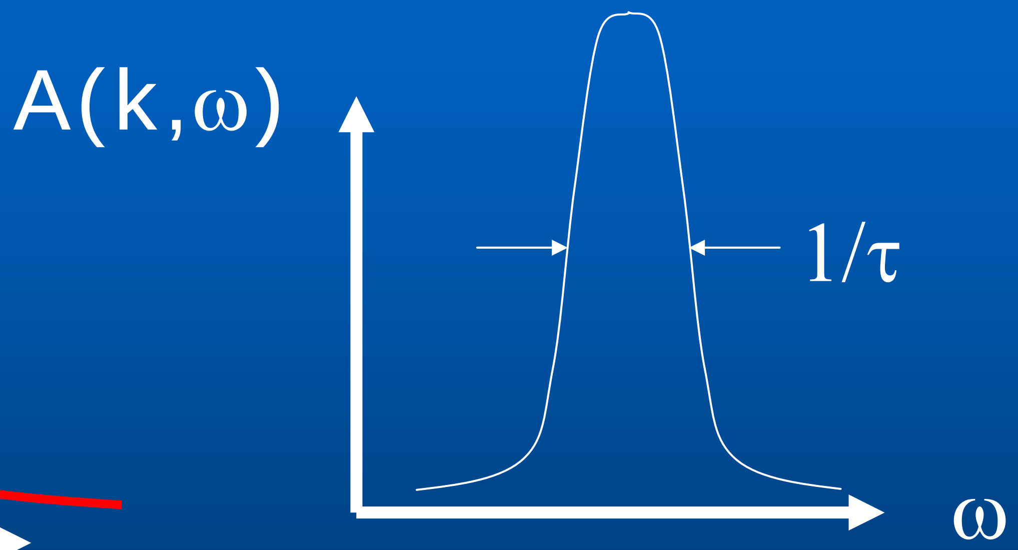


(M. Grioni)

This is *crazy* !



$U \sim 1$ eV



E(k)

$1/\tau \sim U$

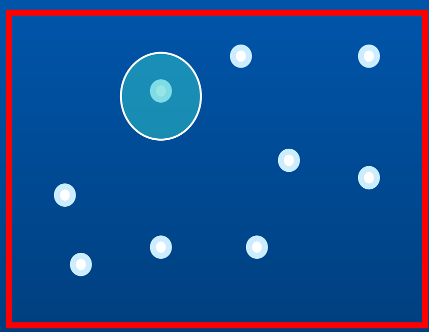
Solution: Fermi liquid theory

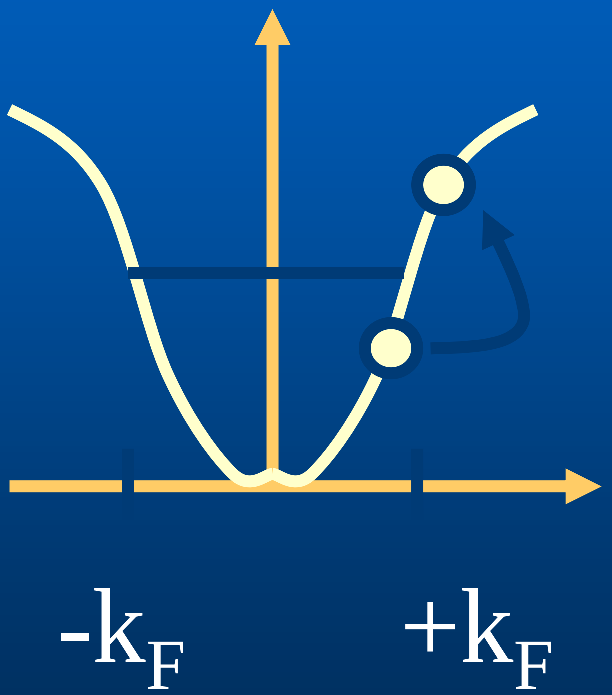
What's up doc !

- Fermi liquids and simple instabilities
- Luttinger liquids
- The dark side...

Fermi liquid 101

- Individual fermionic excitations exist (as for free electrons)





$$m \rightarrow m^*$$

- dressed electron (quasiparticles)

Quasiparticles are *not* free

$$H = \frac{1}{2}\sum f_{p\sigma,p'\sigma'} n_{p\sigma} n_{p'\sigma'}$$

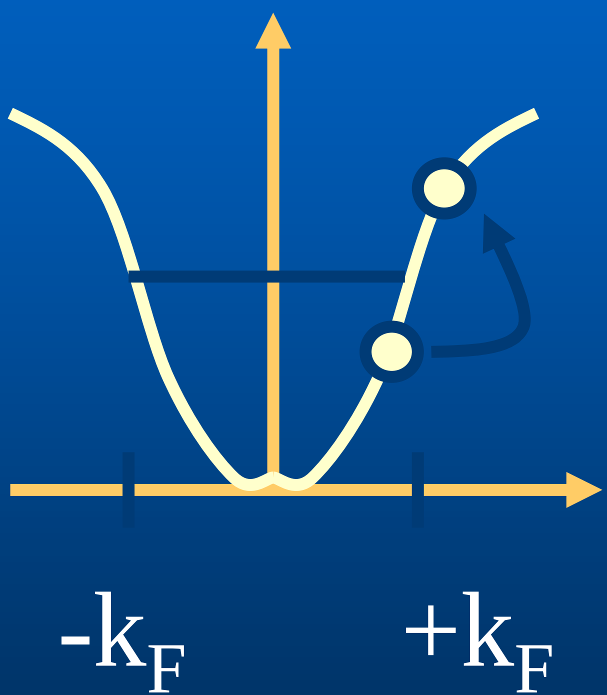
$$\chi \to \chi^*$$

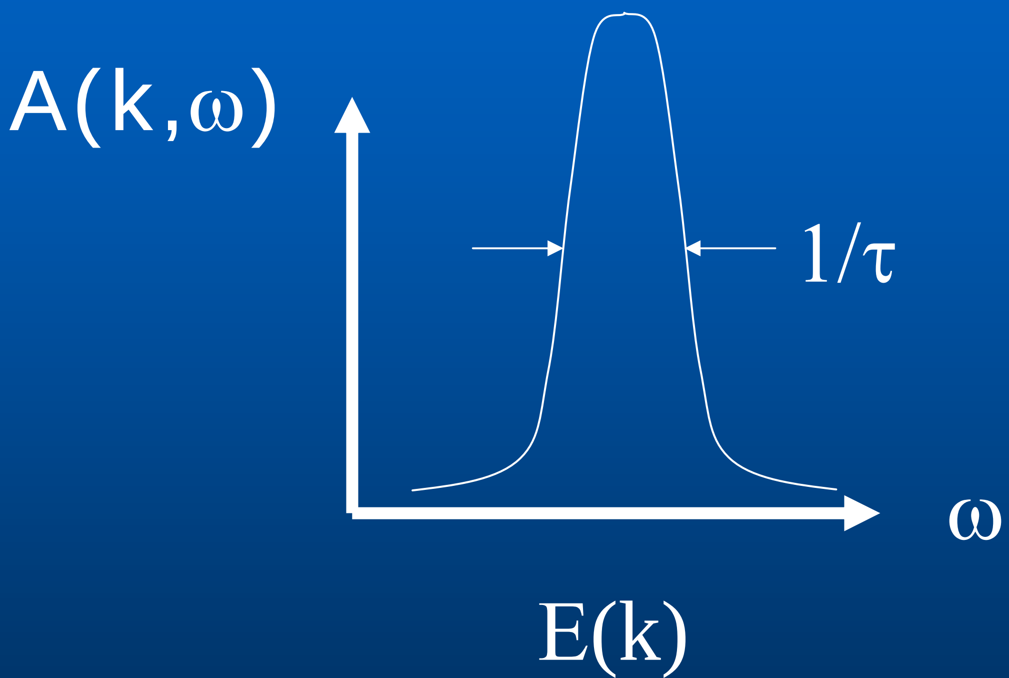
$$\kappa \longrightarrow \kappa^*$$

Landau's parameters

Lifetime

- scattering between QP: lifetime





$$N = \sum_{k1,k2} = N(0)^2 \int_0^{\omega} d\epsilon_1 \int_{\epsilon_1-\omega}^{0} d\epsilon_2 = \frac{\omega^2}{2}$$

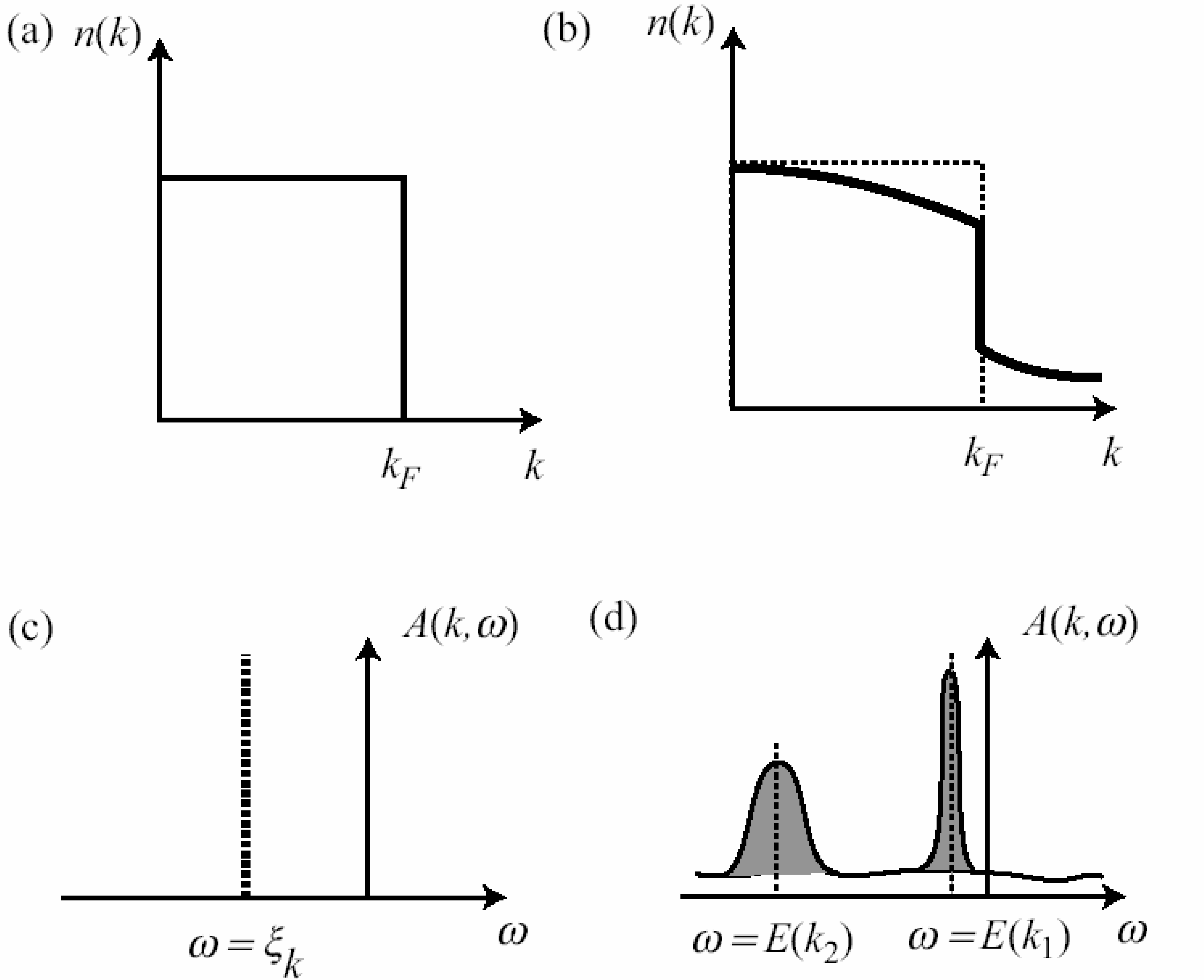
- Lifetime larger than average energy

$$\psi \propto e^{iE(k)t - t/\tau} \qquad 1/\tau \propto \omega^2$$

- QP are sharp (nearly free) excitations close to the Fermi surface

- Transport: $\rho \sim T^2$

- Only a fraction Z in QP states



(a)
$n(k)$
k_F
k
(b)
$n(k)$
k_F
k
(c)
$A(k,\omega)$
$\omega=\xi_k$
ω
(d)
$A(k,\omega)$
$\omega=E(k_2)$
$\omega=E(k_1)$
ω

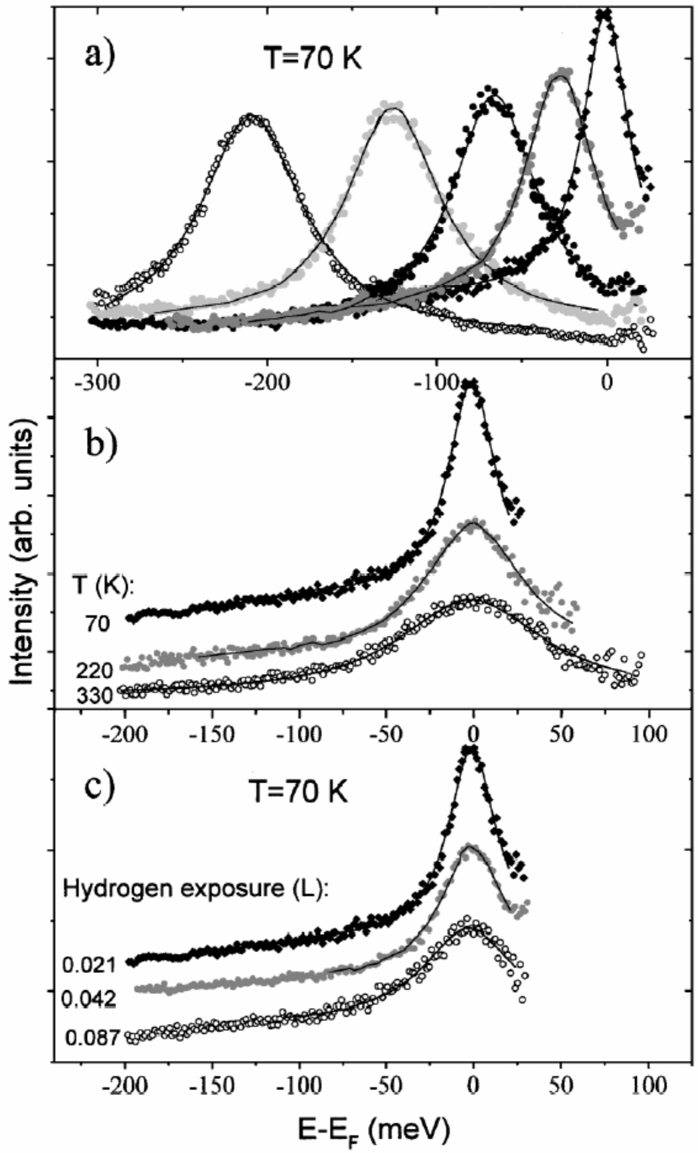


FIG. 2. Spectral intensity as a function of binding energy for constant emission angle, normalized to the experimentally determined Fermi cutoff. Data are symbols, while lines are fits to the Lorentzian peaks with a linear background. The dependence on the (a) binding energy, (b) temperature, and (c) hydrogen exposure is shown.

T. Valla et al. PRL 83 2085 (1999)

Mo(110) surface

More formal

$$A(k,\omega) = \frac{-1}{\pi} \mathrm{Im}\, G_{\mathrm{ret}}(k,\omega)$$

$$\langle \psi(k,\omega)\psi^\dagger(k,\omega) \rangle$$

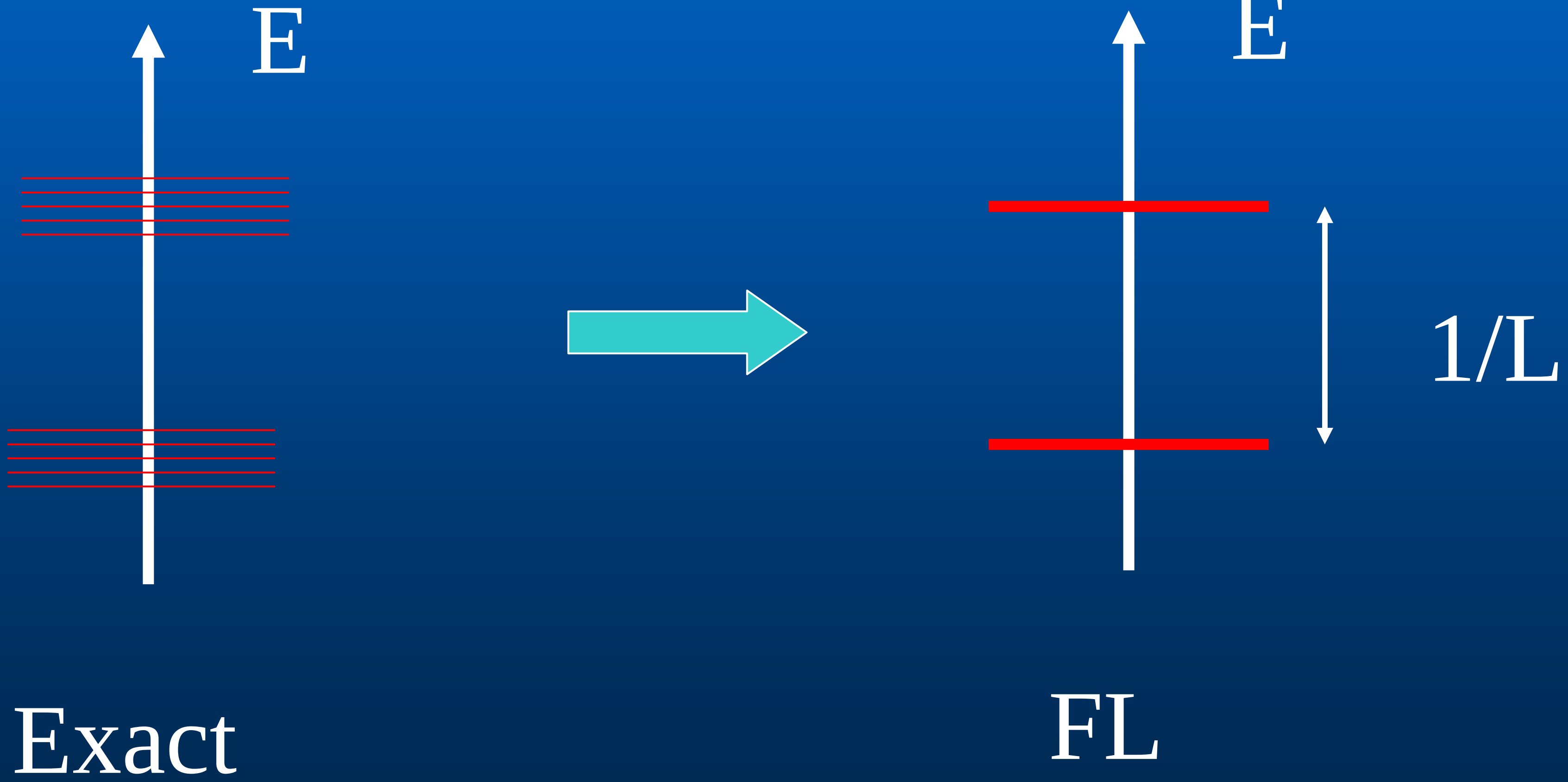
$$G^0_{\mathrm{ret}}(k,\omega) = \frac{1}{\omega - \epsilon(k) + i\delta}$$

$$G_{\mathrm{ret}}(k,\omega) = \frac{1}{\omega - \epsilon(k) - \Sigma(k,\omega)}$$

$\mathrm{Re}\Sigma \rightarrow m^*, Z$

$\mathrm{Im}\Sigma \rightarrow 1/\tau$

- Number of states: exp(Ω)
- Quasiparticles : Δ ~ 1/L



Collective modes

- Charge mode:

  Zero sound: $\omega = V_\rho q$

  Plasmon: $\omega = \mathrm{Cste}$

- Spin mode:

  Spin wave: $\omega = V_s q$

Summary of Fermi liquid

- Thermodynamics:

$$C_V \propto T \qquad \kappa = Cste$$

- « Individual » fermionic excitations

(effective mass m*, weight Z)

- Lifetime: transport etc. $\rho \propto T^2$

- Collective modes (charge and spin)

Fermi liquid theory

- Shown perturbatively in U
- Much more general and robust

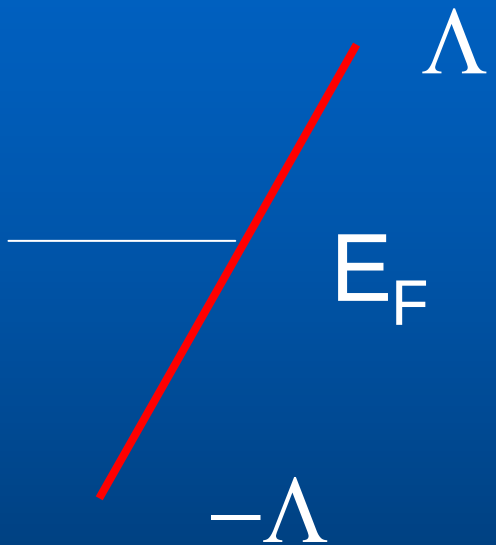
Element	m*/m	χ/χ_0
Nb	2	1
3He	6	20
Heavy fermion	100	100

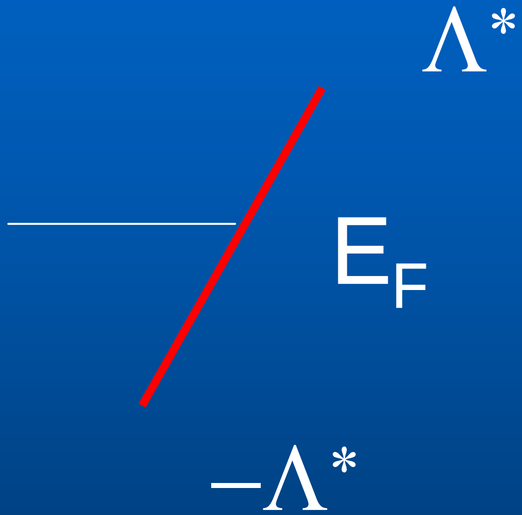
How to prove FL theory

- Perturbatively in U (diagrams)
- Self consistent proof:

  discontinuity at Fermi surface is an order parameter.
- RG picture (perturbative)

RG interpretation of FL





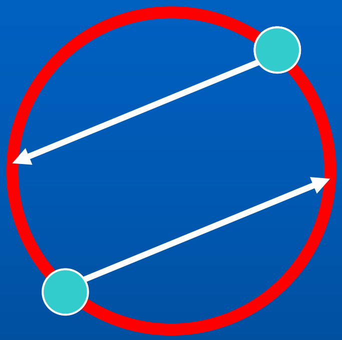
Λ: cutoff

U: interaction

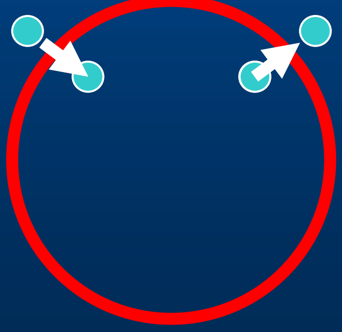
Λ^*: cutoff

U^*: interaction

Hamiltonian (Landau)



$$H_{\text{int}} = \sum_{k,k',q} V(q) \quad c^*_{k+q}\, c^*_{k'-q}\, c_{k'} c_k$$



$$H_{FL} = \sum_{k,k'} f_{\hat{k},\hat{k}'} n(k) n(k')$$

Use of Fermi liquids

- Takes care of largest and most complicated processes
- Starting point to take into account perturbations (lattice, impurities, etc.)
- Simple instabilities and ordered states

How to break this boring stuff

- Strong or unusual interactions (localized electrons, BCS, …)

- Special fermi surfaces (nesting, singularities at E_F)

- Special dimensions (d=1, d=2 (?))

Nesting

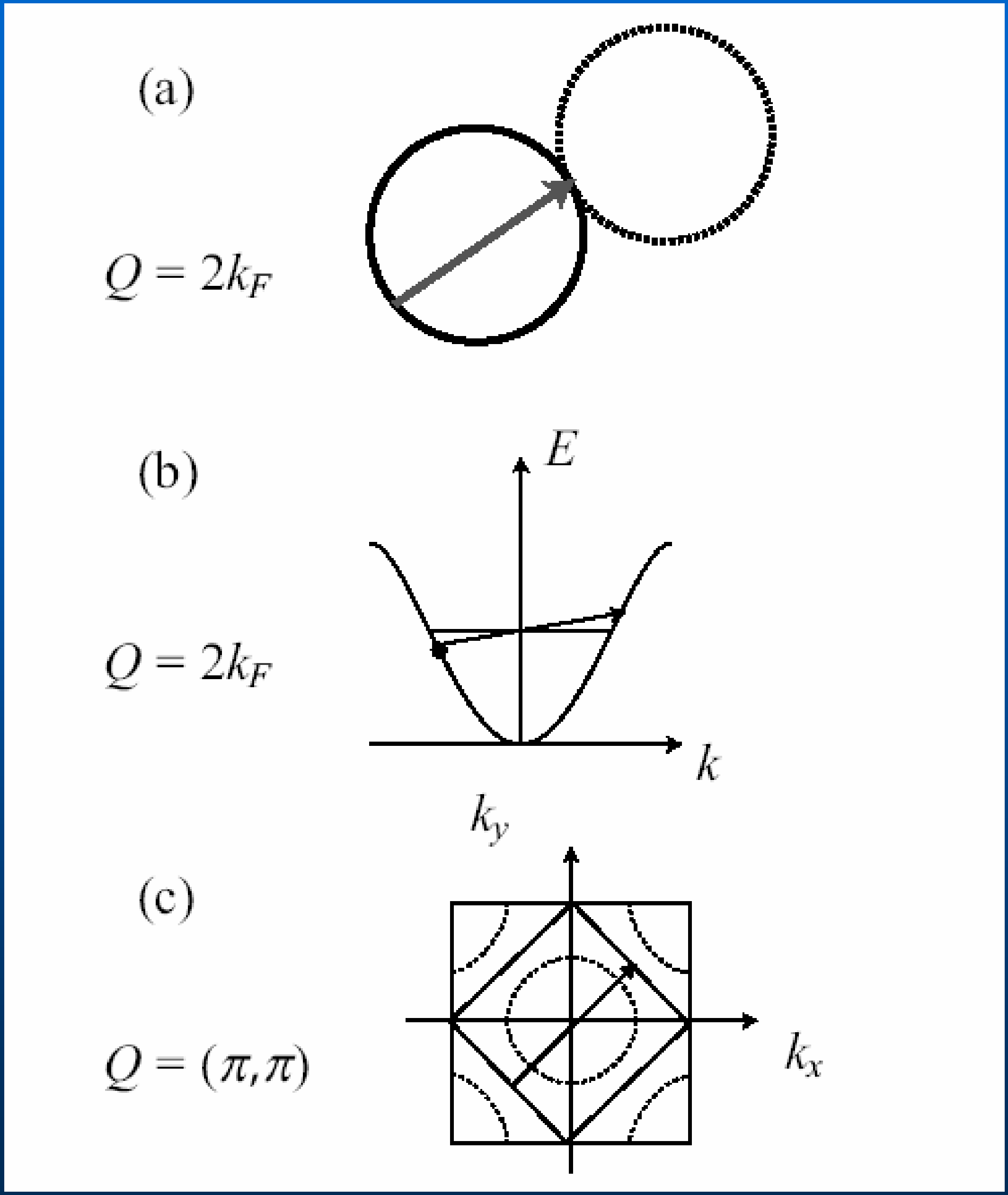
$$\chi(q,\omega) = \frac{1}{\Omega}\sum_k \frac{f_F(\xi_k) - f_F(\xi_{k+q})}{\omega + \xi(k) - \xi(k+q) + i\delta}$$

$$\xi(k+Q) = -\xi(k)$$

$$\mathrm{Re}\ \chi(Q,\omega=0) = -\int d\xi\ N(\xi)\frac{\tanh(\beta\xi/2)}{2\xi}$$

$$\chi(Q,\omega=0) \sim -N(\xi=0)\ \log(E/T)$$

Divergent !



Difficult in high dimensions

Easy in d=1

Special surfaces

What happens ?

Naively : ordered state

$$\chi(q,\omega) = \frac{\chi^0(q,\omega)}{1+U\chi^0(q,\omega)}$$

RPA

$\chi = \infty$ at T_c

Instabilities compete

Fluctuations ?

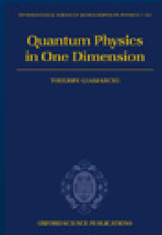
How to *really* break FL

- Lower dimension (fluctuations)
- Increase interactions
- Strange degrees of freedom

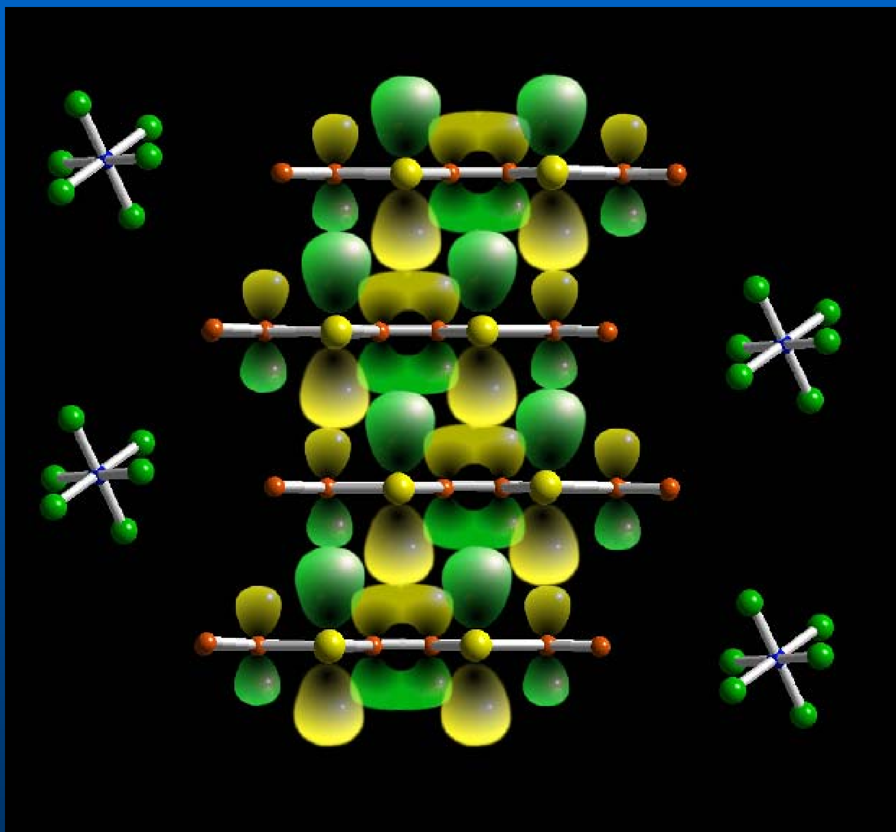
Luttinger liquids

References on 1D Physics

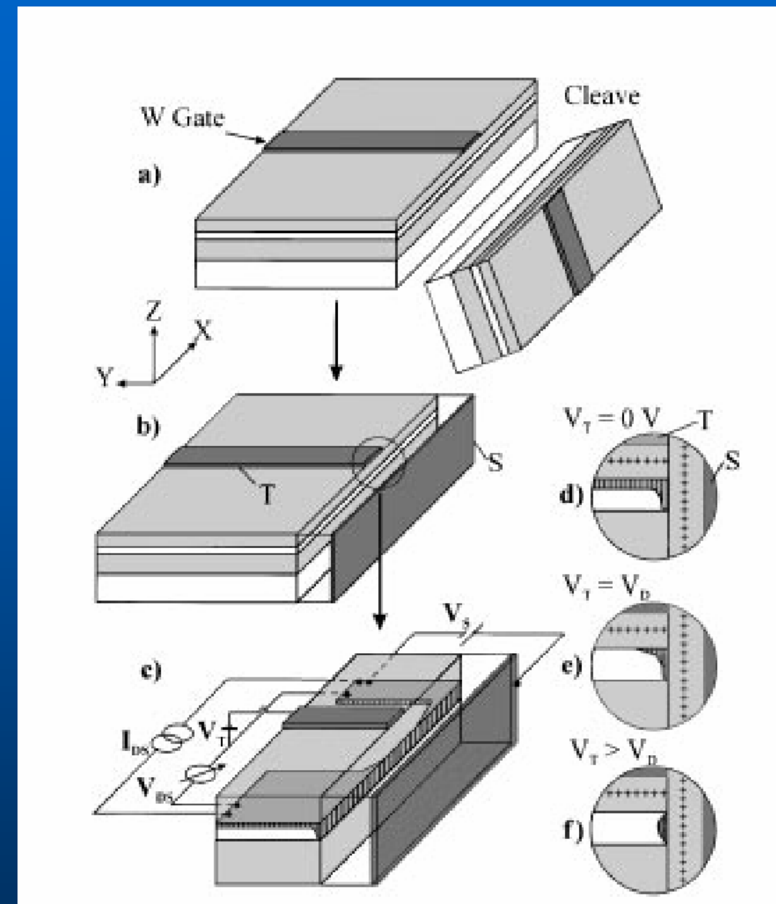
- Emery, V. J. (1979). Highly conducting one dimensional solids, pp. 247. Plenum.
- Solyom, J. (1979). Adv. Phys., 28, 209.
- Schulz, H. J. (1995). *Les Houches LXI* pp. 533. Elsevier.
- Voit, J. (1995). Rep. Prog. Phys., 58, 977.
- Senechal, D. (2003). CRM Series in Mathematical Physics, Springer.
  cond-mat/9908262.
- Gogolin, A. O., Nersesyan, A. A., and Tsvelik, A. M. (1999). Bosonization and Strongly Correlated Systems. Cambridge University Press, Cambridge.
- Giamarchi, T. (2004) Quantum Physics in one dimension. Oxford University Press, Oxford.



1d Systems

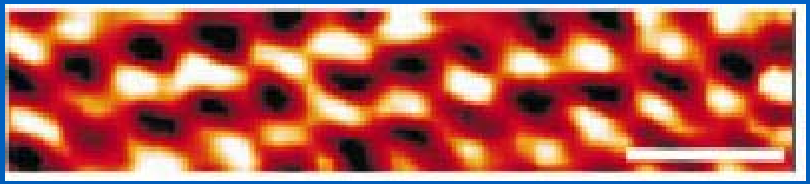


Organic conductors

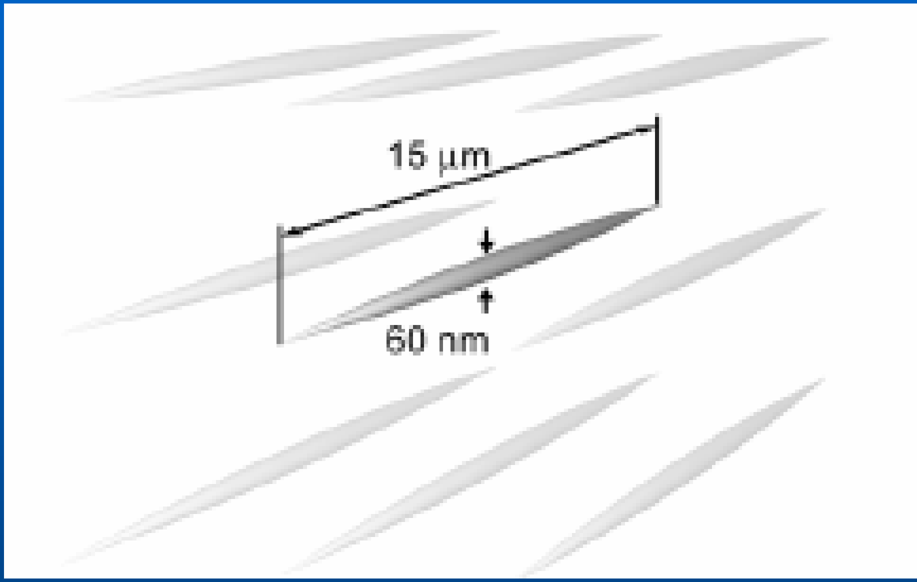


Quantum wires

Cold Bosons



Nanotubes



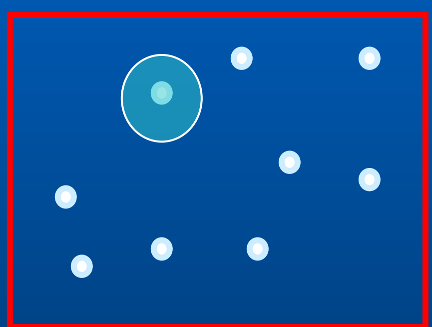
But also:
josephson junctions
ladders
Edge states …..

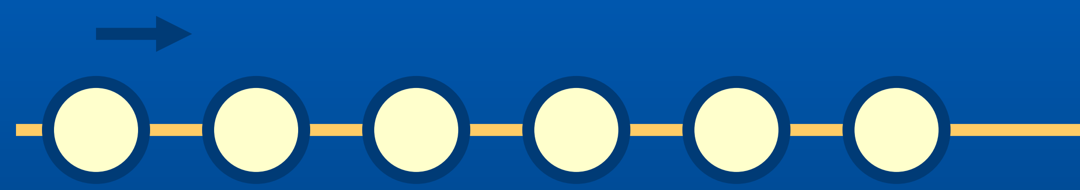
$N_0 \sim 10$ to 10^3 atoms

T. Stoferle *et al.* PRL **92** 130403 (2004)

What is special in d=1 ?

- No individual excitation can exist (only collective ones)

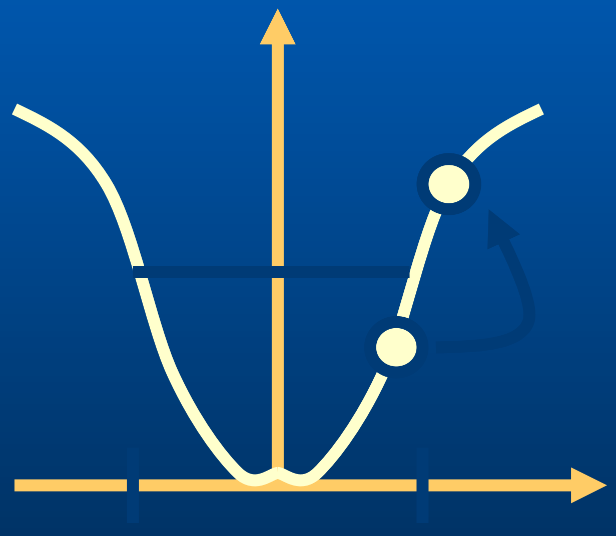


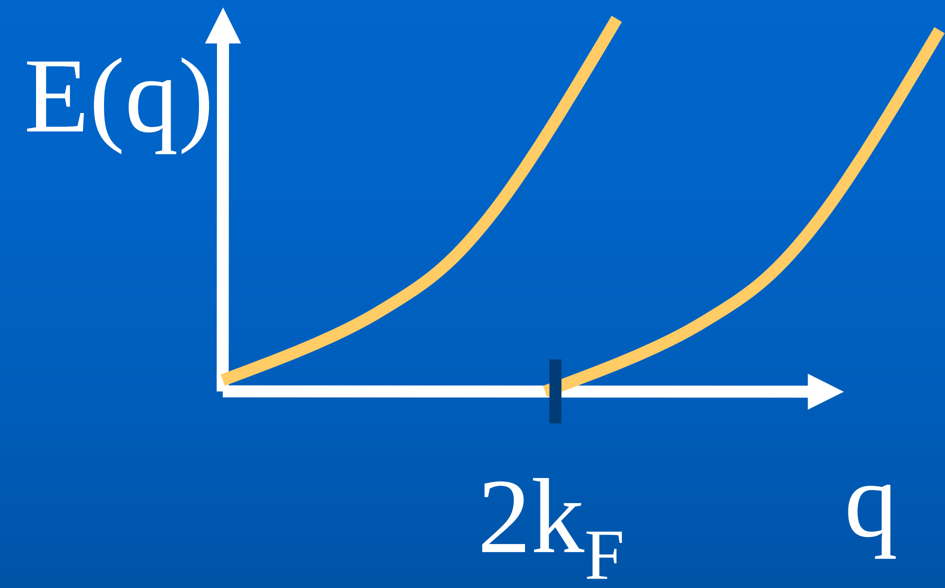


- Strong quantum fluctuations (``no’’ ordered state or mean field possible)

What are the good excitations ?

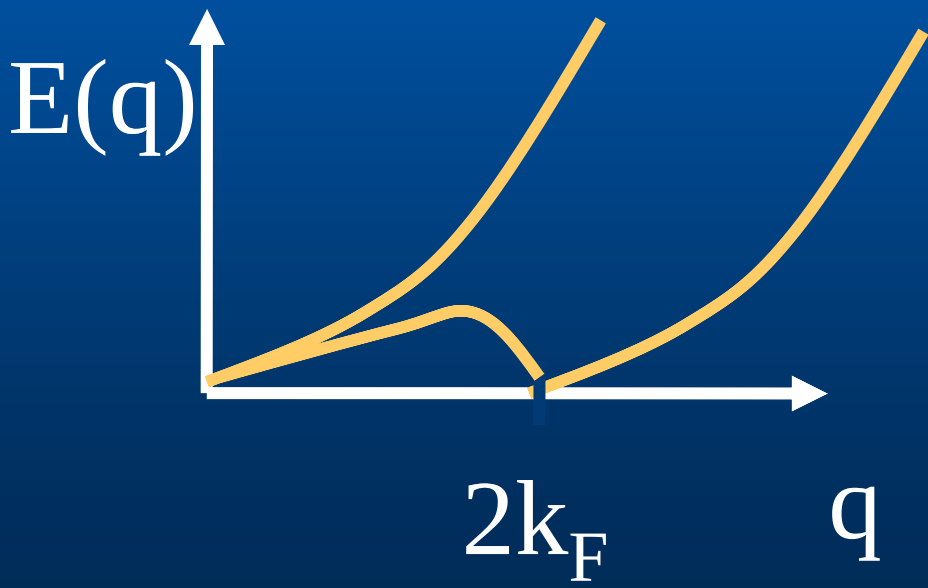
$$E(k,q) = \varepsilon(k+q) - \varepsilon(k)$$





D>1:

continuum

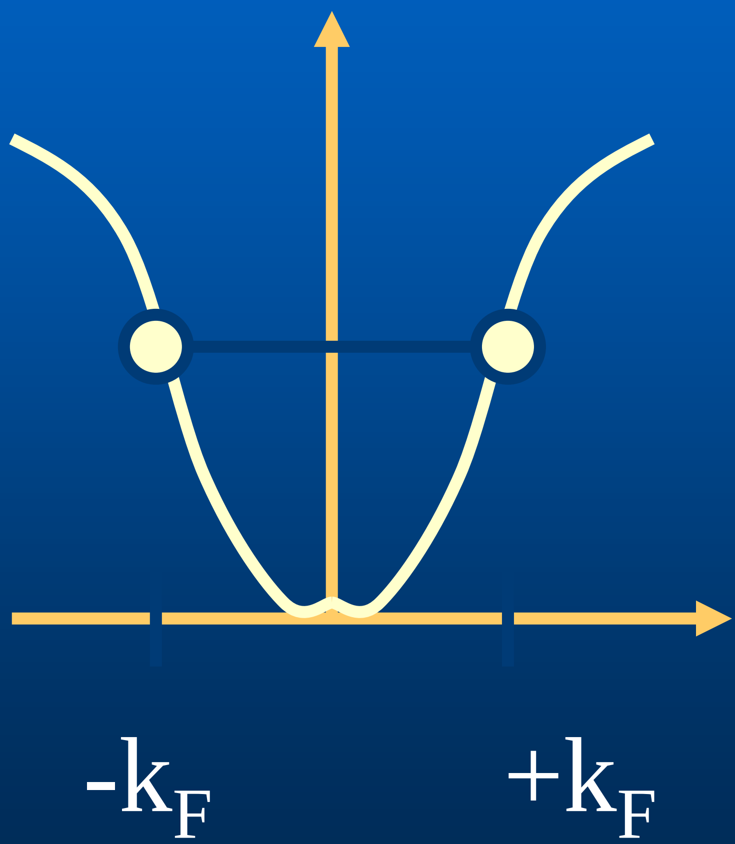


D=1:

Well defined excitations

$$E(q) = v_F q$$

Nesting is the rule



Nesting

$$\varepsilon(k+Q) = -\varepsilon(k)$$

$Q = 2\ k_F$

How to study

- Exact methods (Bethe Ansatz)

  Exact

  spectrum; limited to very special models

- Numerics

  ``Exact''

  special models, size limitations,

  quantities specific to models

- Low energy methods

Reexpress the Kinetic energy

$$\nabla\Phi(x) = -\pi[\rho_R(x) + \rho_L(x)]$$

$$\nabla\Theta(x) = \pi[\rho_R(x) - \rho_L(x)] = \pi\Pi(x)$$

$$H = \int \frac{dx}{2\pi} v_F [(\pi\Pi(x))^2 + (\nabla\Phi(x))^2]$$

- ``Phonon'' Hamiltonian (sound waves of charge)

Interactions

$$\rho(x)\rho(x') \approx (\nabla\Phi(x))^2$$

$$H = \int \frac{dx}{2\pi}[uK(\pi\Pi(x))^2 + \frac{u}{K}(\nabla\Phi(x))^2]$$

- u velocity of sound
- K dimensionless parameter,

K< 1 : repulsive

K>1 : attractive

Properties

- Only collective excitations

- Thermodynamics

$$C_V = \frac{T}{u}\left(\frac{L\pi}{3}\right) \qquad \kappa / \kappa_0 = K \frac{v_F}{u}$$

- Looks like a Fermi liquid for q~0

Correlation functions

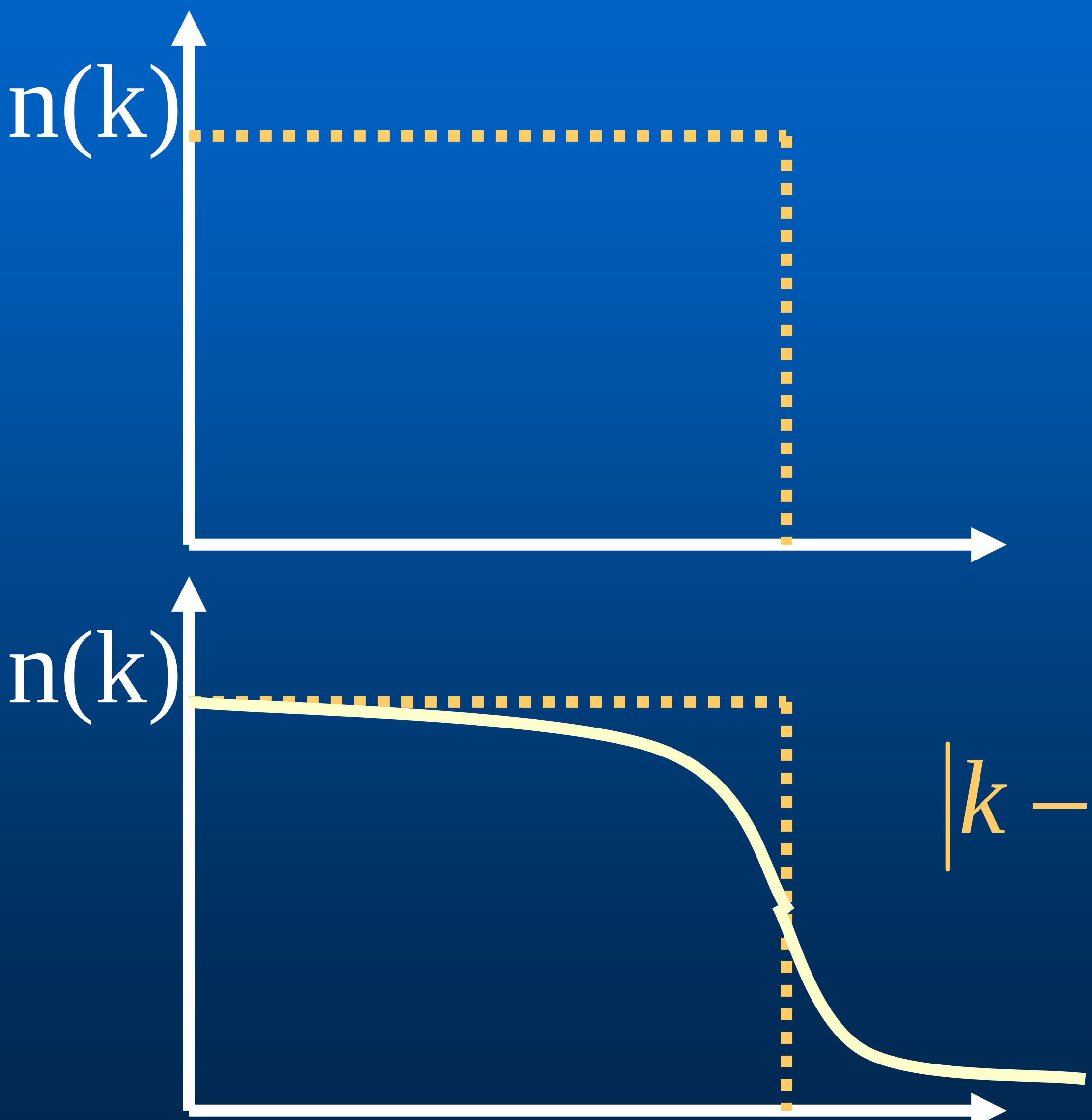
$$\left\langle \rho(x)\rho(0)\right\rangle = \frac{1}{x^2} + \cos(2k_F x)\left(\frac{1}{x}\right)^{2K}$$

$$\left\langle O_{SU}(x) O_{SU}(0)\right\rangle = \left(\frac{1}{x}\right)^{1/2K}$$

$$\left\langle \psi_R(x)\psi_R^*(0)\right\rangle = \left(\frac{1}{x}\right)^{\frac{1}{2}[K+K^{-1}]} e^{iArg(\tau/x)}$$

$$K = 1 \qquad \left\langle \psi_R(x)\psi_R^*(0)\right\rangle = \frac{1}{x - v_F \tau}$$

No Landau Quasiparticles

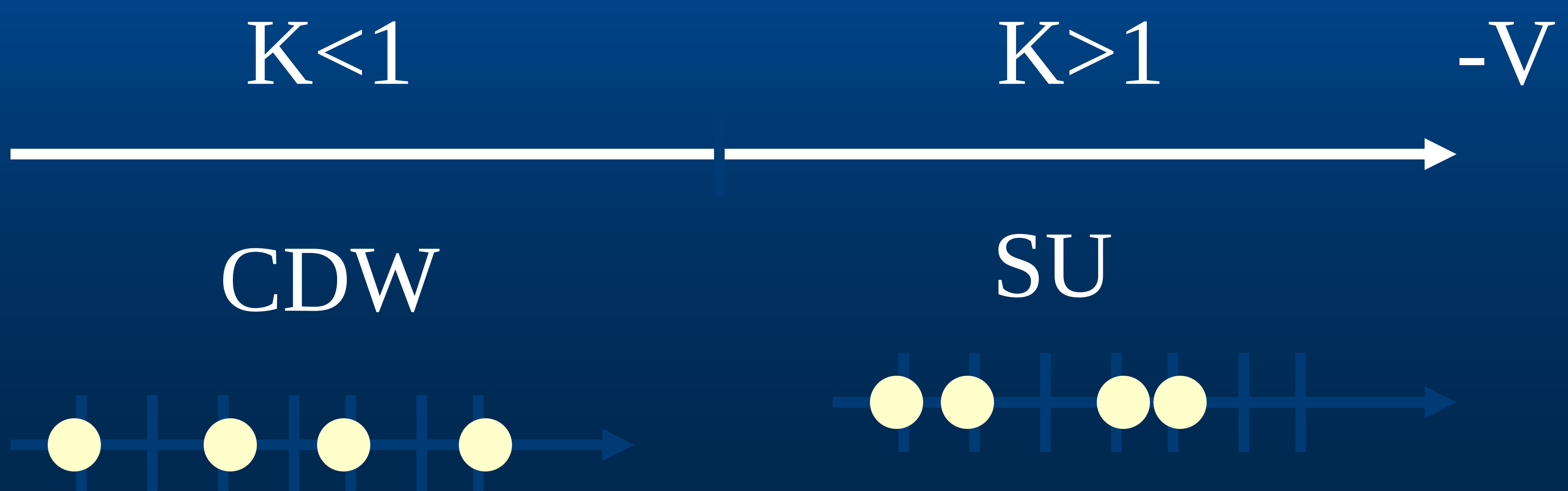


$$\left|k-k_F\right|^{\frac{1}{2}\left[K+K^{-1}\right]-1}$$

Phase diagram

$$\chi(q,\omega) = \int dx\,d\tau\; e^{i(qx+\omega\tau)}\chi(x,\tau) \qquad \chi \approx \omega^{\eta-2}$$

Most divergent fluctuations



System with spin

Same treatment

$$\rho_{\uparrow} \rightarrow \nabla\Phi_{\uparrow} \qquad \rho_{\downarrow} \rightarrow \nabla\Phi_{\downarrow}$$

More convenient

$$\rho = \frac{1}{\sqrt{2}}(\rho_{\uparrow} + \rho_{\downarrow}) \qquad \sigma = \frac{1}{\sqrt{2}}(\rho_{\uparrow} - \rho_{\downarrow})$$

$$H_{kin} = H_{\uparrow} + H_{\downarrow} = H_{\rho} + H_{\sigma}$$

$$H_{\text{int}} = U \sum_i \rho_\uparrow \rho_\downarrow = U(\rho + \sigma)(\rho - \sigma)$$

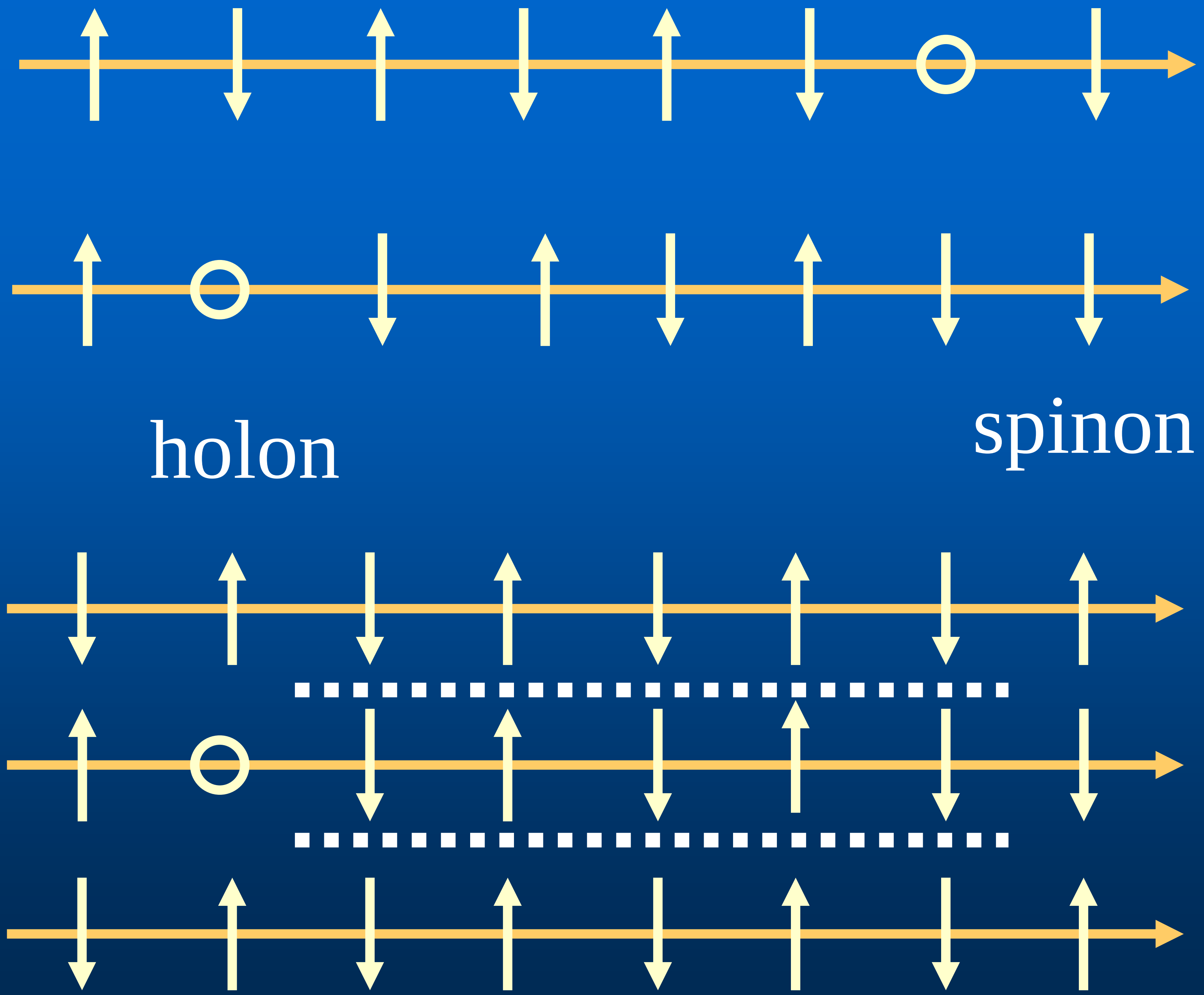
$$= U(\rho\rho - \sigma\sigma)$$

$$H = H_\rho + H_\sigma$$

(u_ρ, K_ρ) Charge excitations

(u_σ, K_σ) Spin excitations

Charge-spin separation


holon
spinon

Correlation functions

$$\langle S(x)S(0)\rangle = \frac{1}{x^2} + \cos(2k_F x)\left(\frac{1}{x}\right)^{K_\sigma + K_\rho}$$

- Perturbation (small U)

$$u_\rho K_\rho = u_\sigma K_\sigma = v_F$$

$$u_\rho / K_\rho = v_F + U/\pi$$

$$u_\sigma / K_\sigma = v_F - U/\pi$$

Spin sector more complicated (gap)

$$H = \int \frac{dx}{2\pi}[uK(\pi\Pi(x))^2 + \frac{u}{K}(\nabla\Phi(x))^2]$$
$$+ g\int dx \cos(\sqrt{8}\Phi(x))$$

Anomalous correlation functions

$$n(k) \approx \left|k - k_F\right|^{\frac{1}{4}[K_\rho + K_\rho^{-1}] - \frac{1}{2}}$$ photoemission

$$\chi_{2k_F} \approx T^{K_\rho - 1}$$ NMR

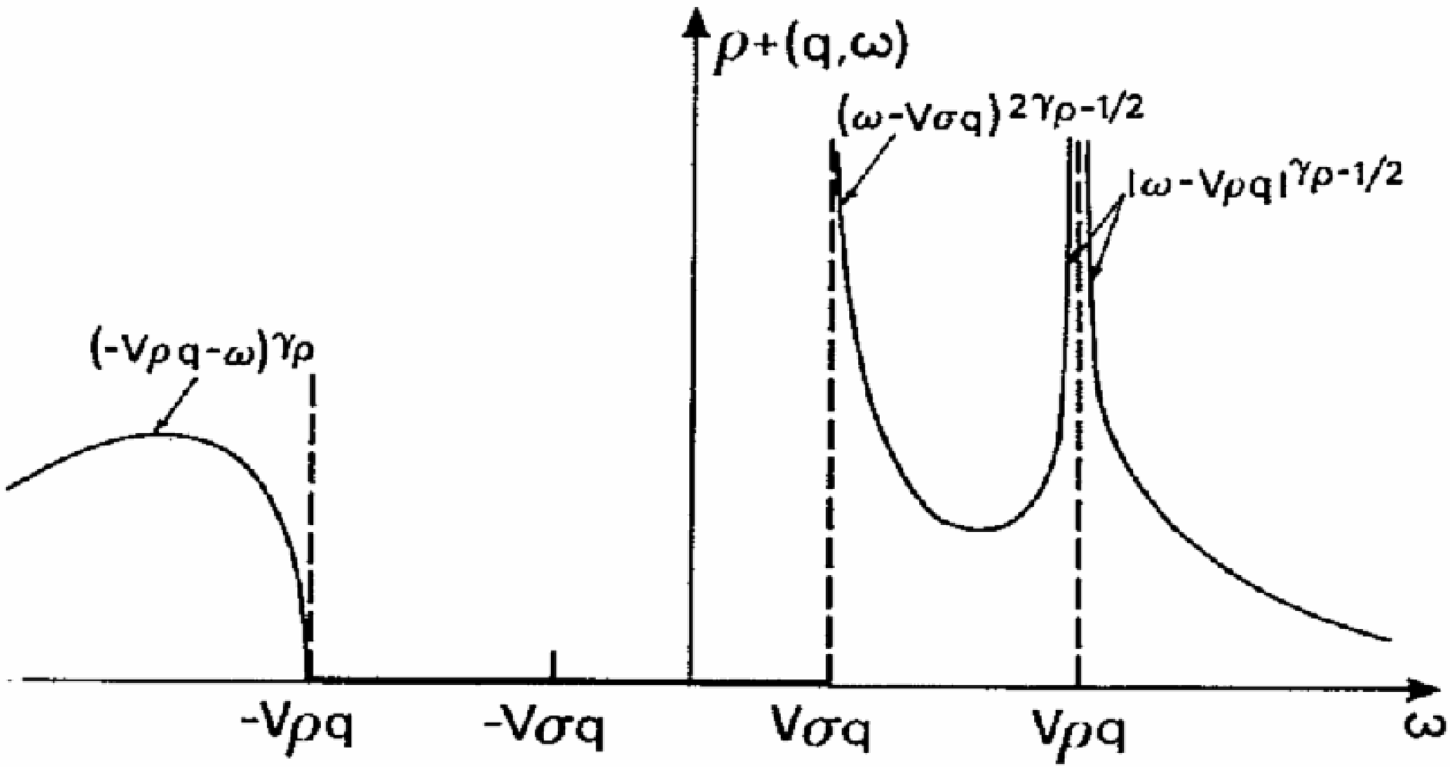
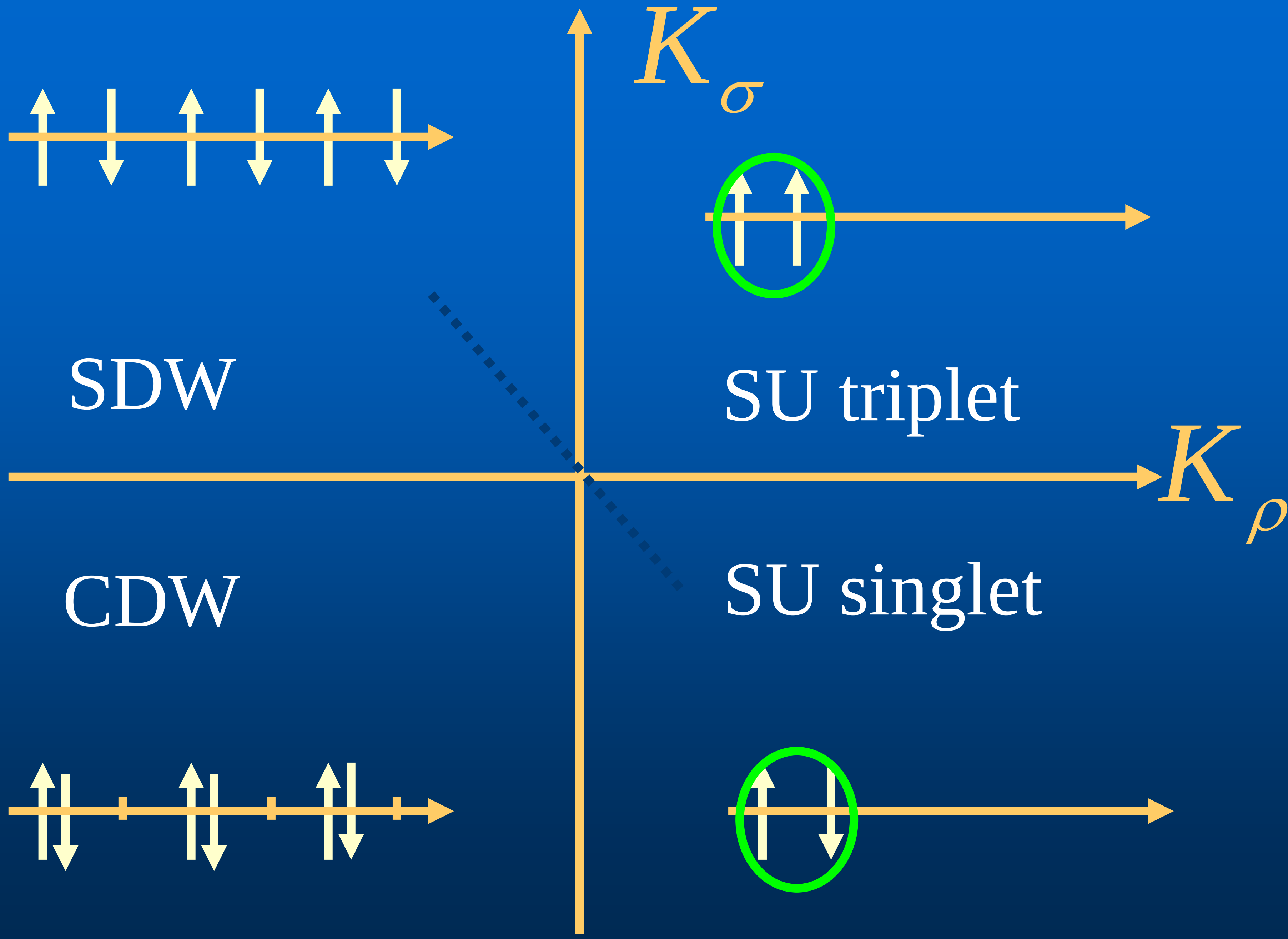


FIG. 3. Spectral function $\rho_+(q,\omega)$ for the spin-$\frac{1}{2}$ Luttinger liquid for $q>0$.

J. Voit PRB 47 6740 (1986)



K_σ
SDW
SU triplet
K_ρ
CDW
SU singlet

Luttinger liquid concept

- How much is perturbative

- Nothing provided the correct u,K are used

- Low energy properties: Luttinger liquid (fermions, bosons, spins…)

Fermions

$$\left\langle \rho(x)\rho(0)\right\rangle = \frac{1}{x^2} + A_1 \cos(2k_F x)\left(\frac{1}{x}\right)^{K_\rho+1} + A_2 \cos(4k_F x)\left(\frac{1}{x}\right)^{4K_\rho}$$

Hubbard

$U = 0 \quad K_\rho = 1$

$U = \infty \quad K_\rho = 1/2$

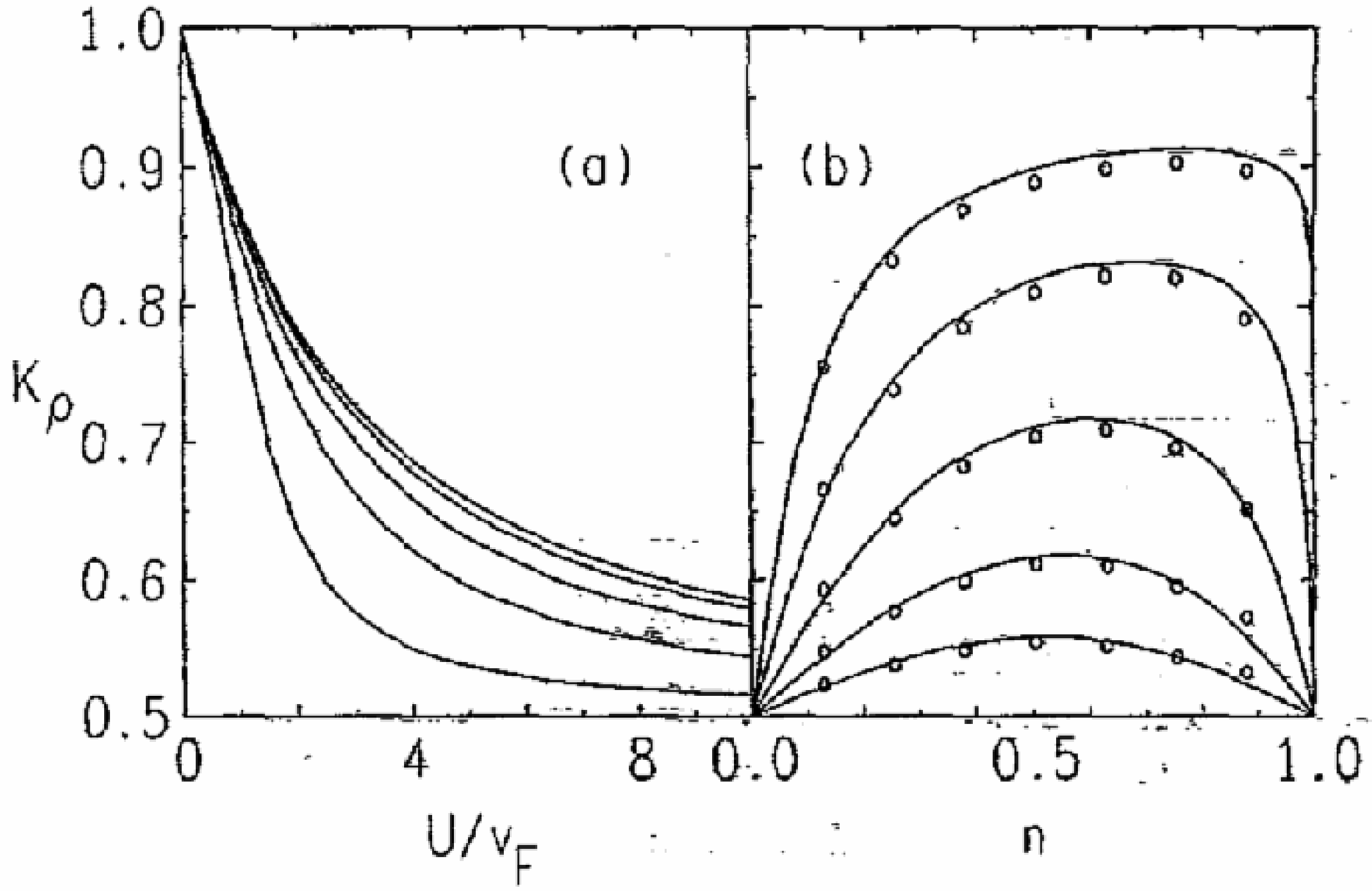
How to compute (u,K)

- Perturbation
- Exact solutions (Bethe Ansatz): thermodynamics

$$C_V \rightarrow u \quad \kappa \rightarrow u/K \quad D = uK$$

$$(E(L) - E(\infty))/L \propto cu$$

- Numerics



H.J. Schulz PRL 64 2831 (90)

Summary of Luttinger liquid

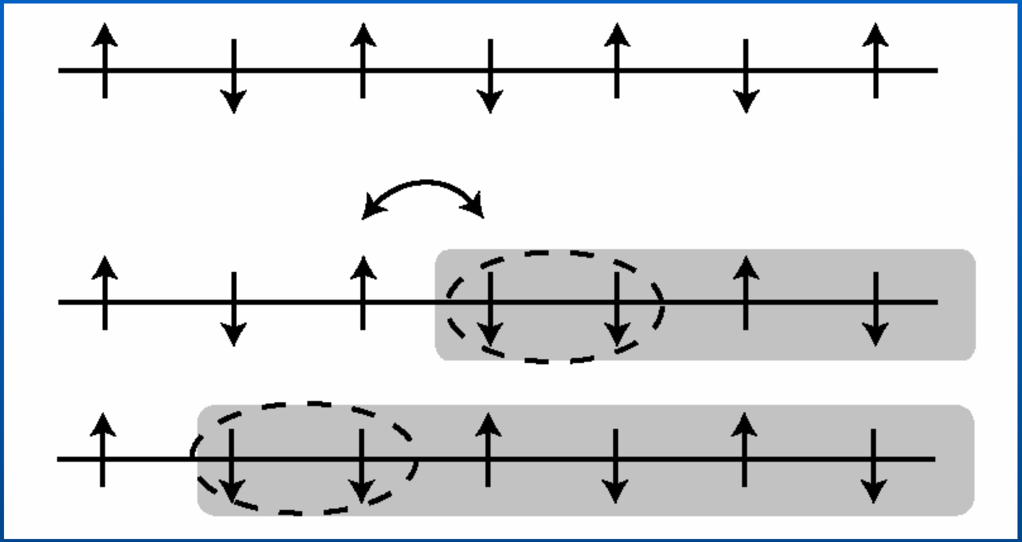
- Spin charge separation

- No fermionic quasiparticles
- Power law decay of correlation functions

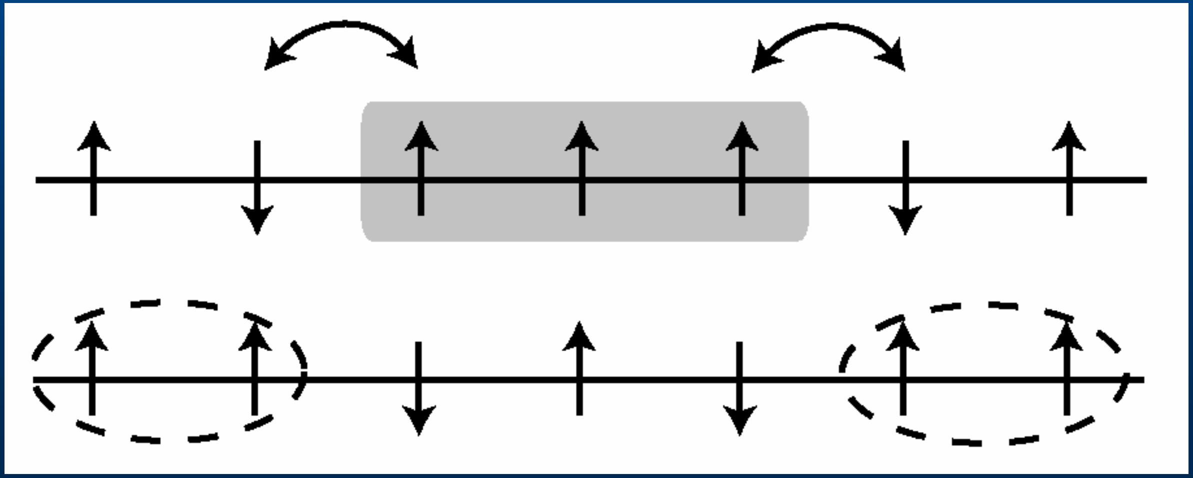
$$\left\langle \rho(x)\rho(0) \right\rangle = \frac{1}{x^2} + \cos(2k_F x)\left(\frac{1}{x}\right)^{1+K_\rho} + \ldots$$

Kρ contains all information about interactions

Fractional ``charges”



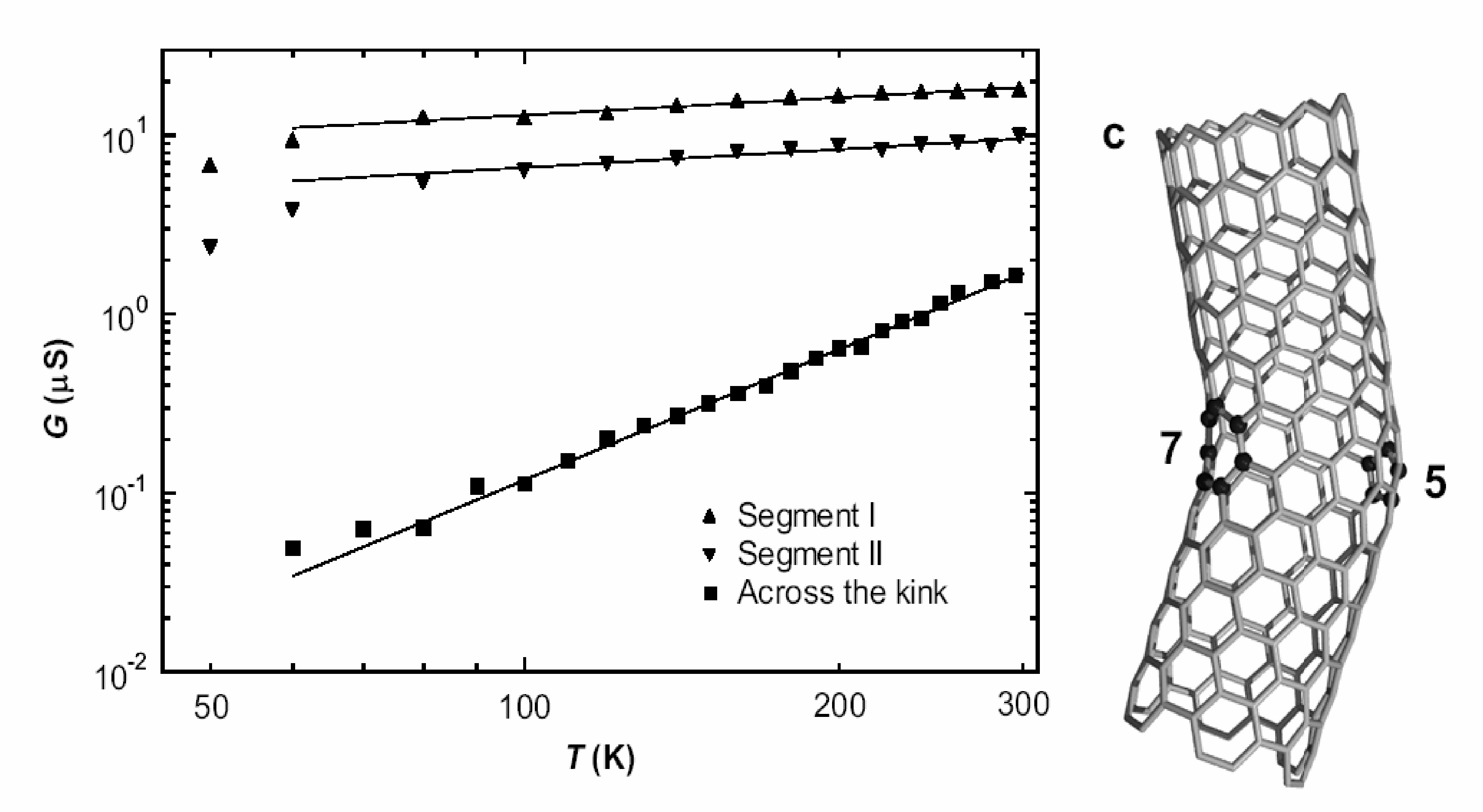
Spinon



Magnon S=1

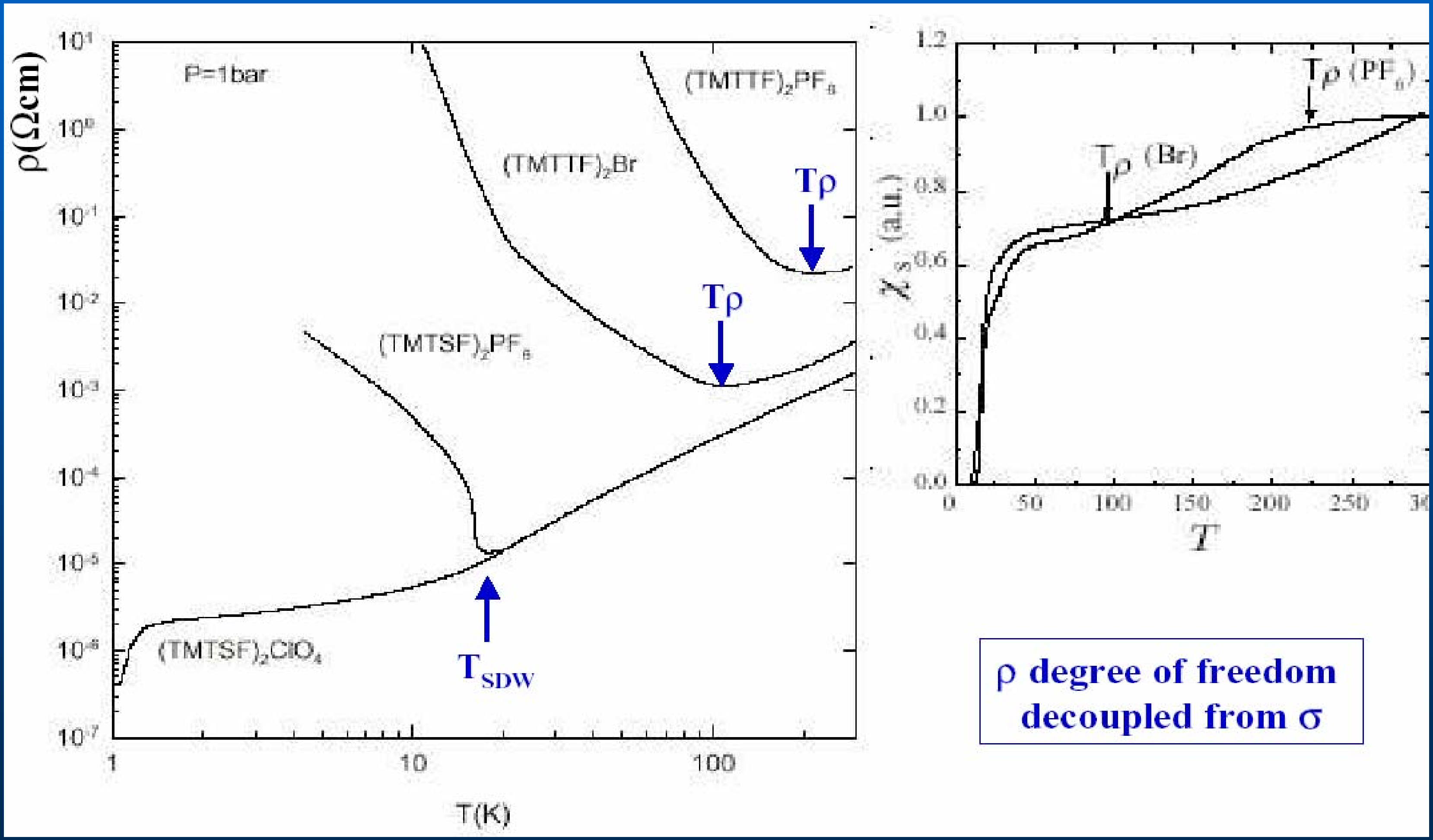
2 Spinons
S=1/2

SP density of states

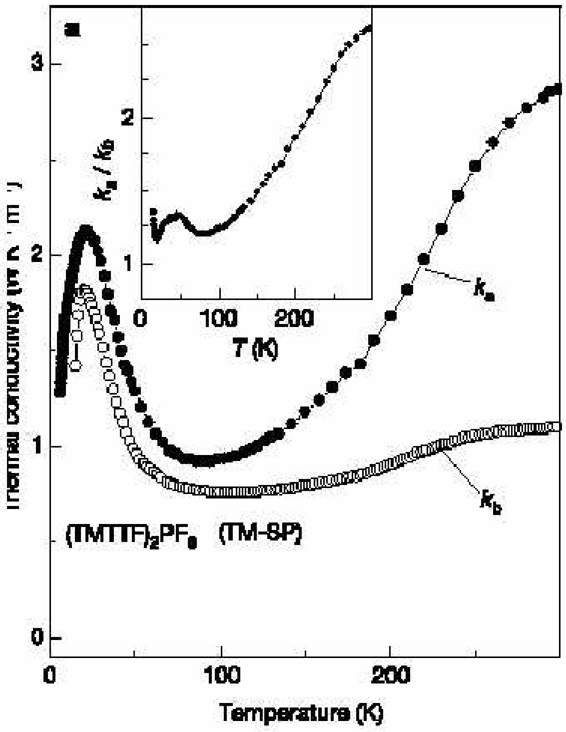


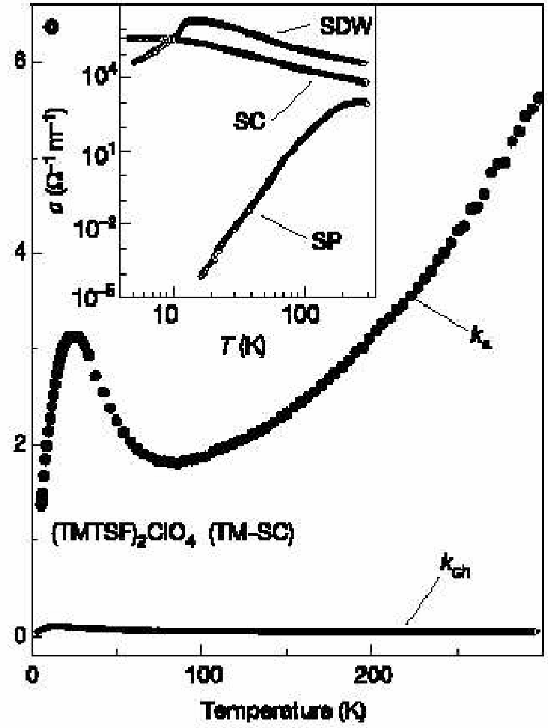
Yao et al., Nature 402 273 (1999)

Spin charge separation ?

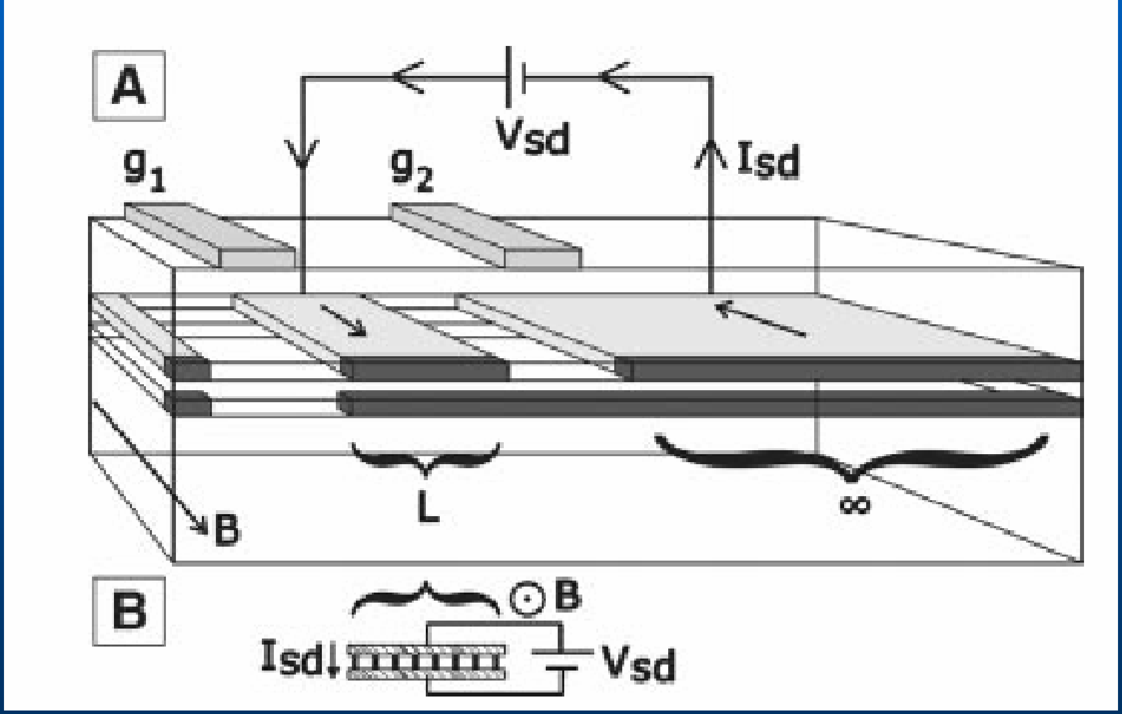


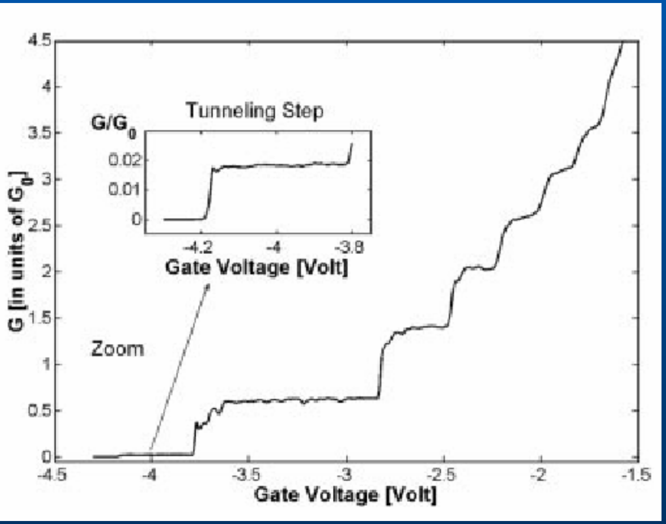
Lorenz et-al, Nature 418, 614, 2002





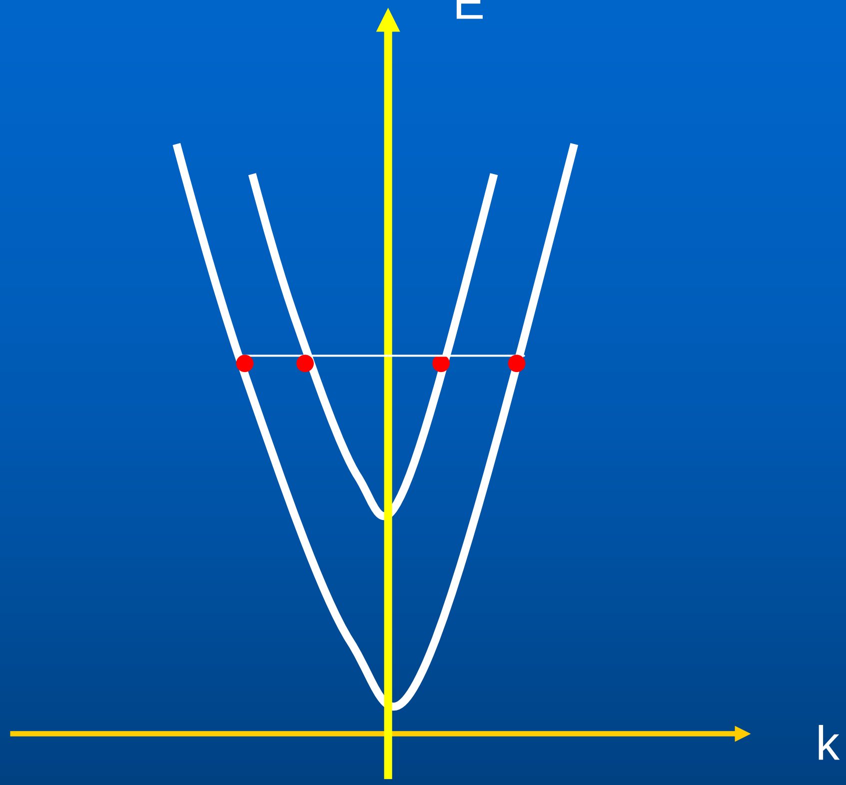
Spin charge separation

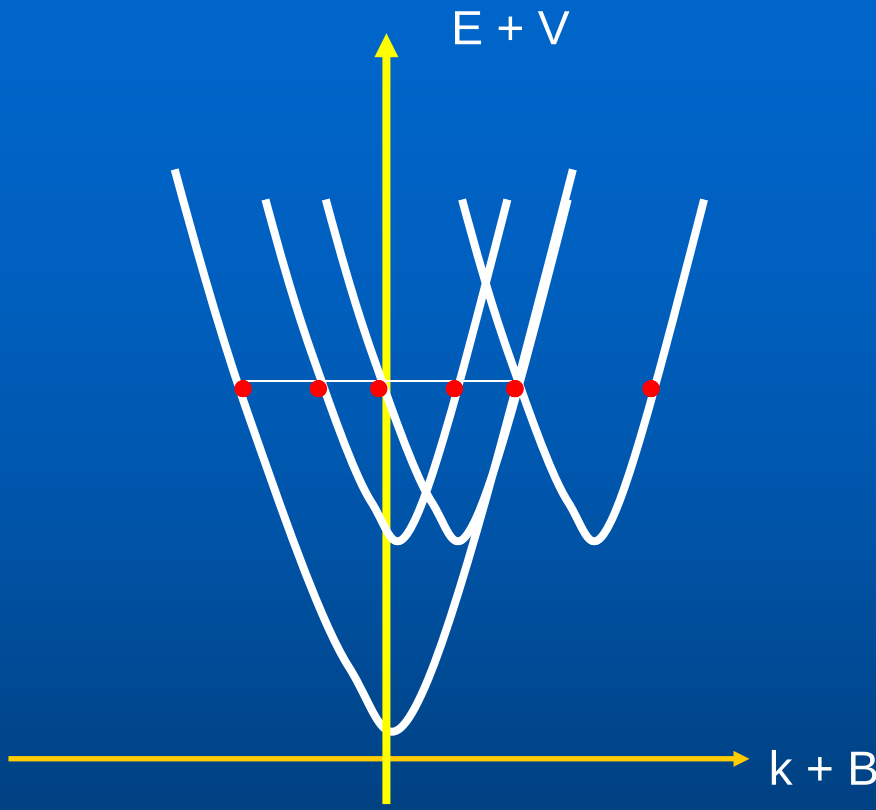


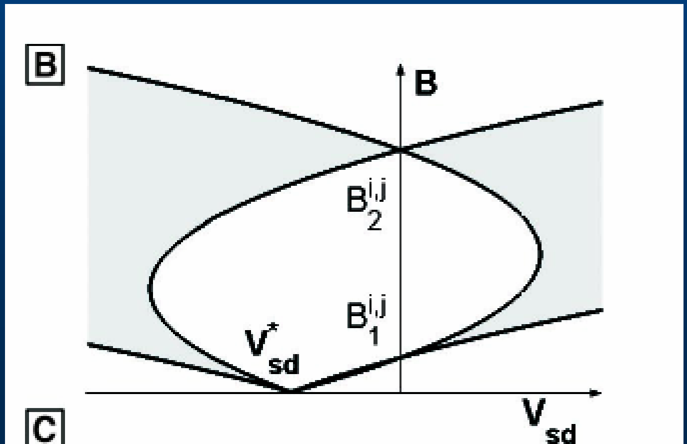


O.M Ausslander et al., Science **298** 1354 (2001)
Y. Tserkovnyak et al., PRL **89** 136805 (2002)
Y. Tserkovnyak et al., PRB **68** 125312 (2003)

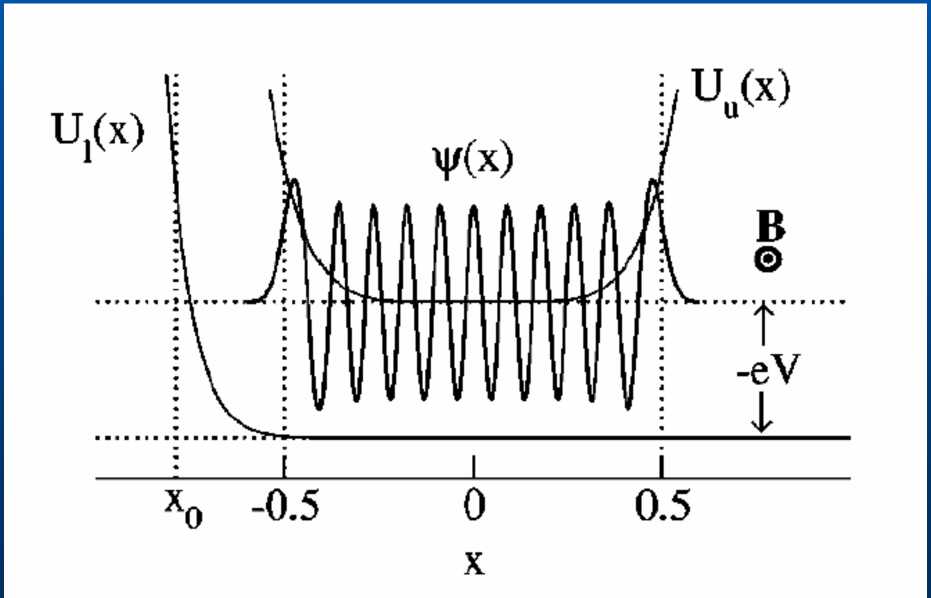
Spectroscopy



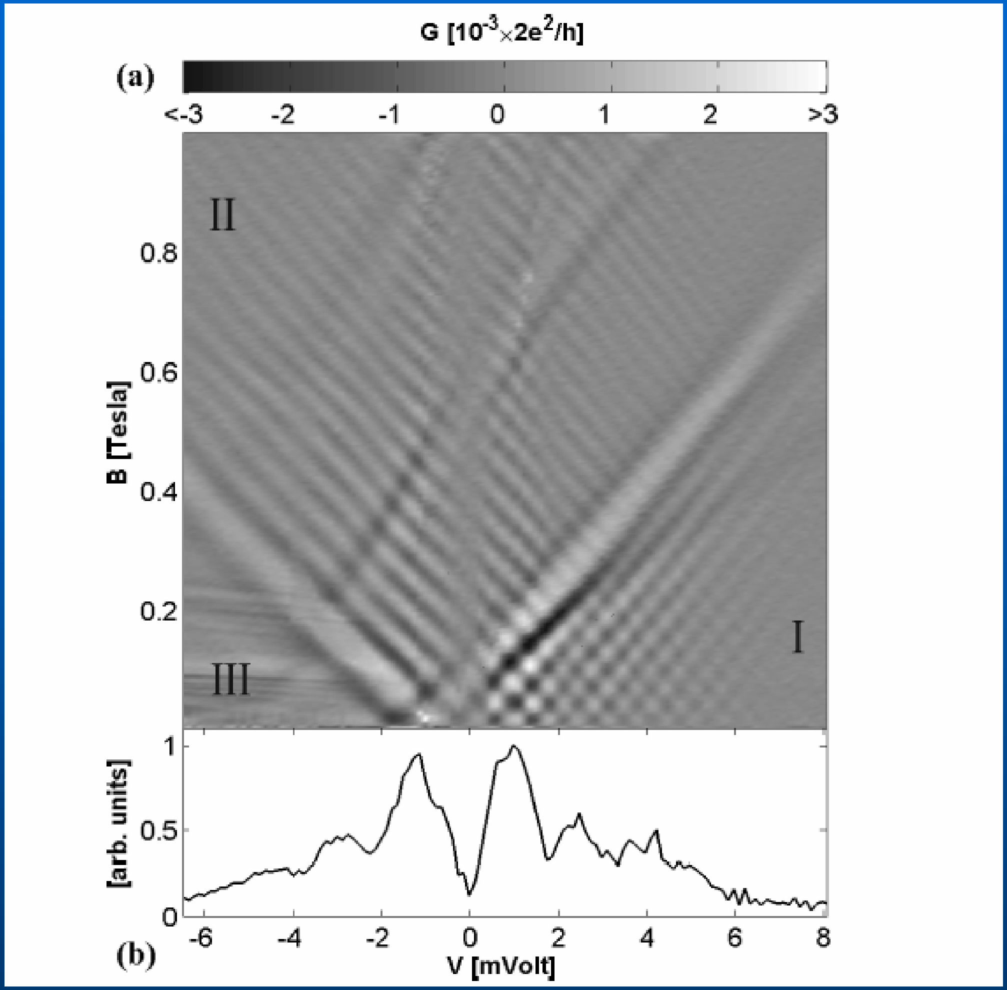


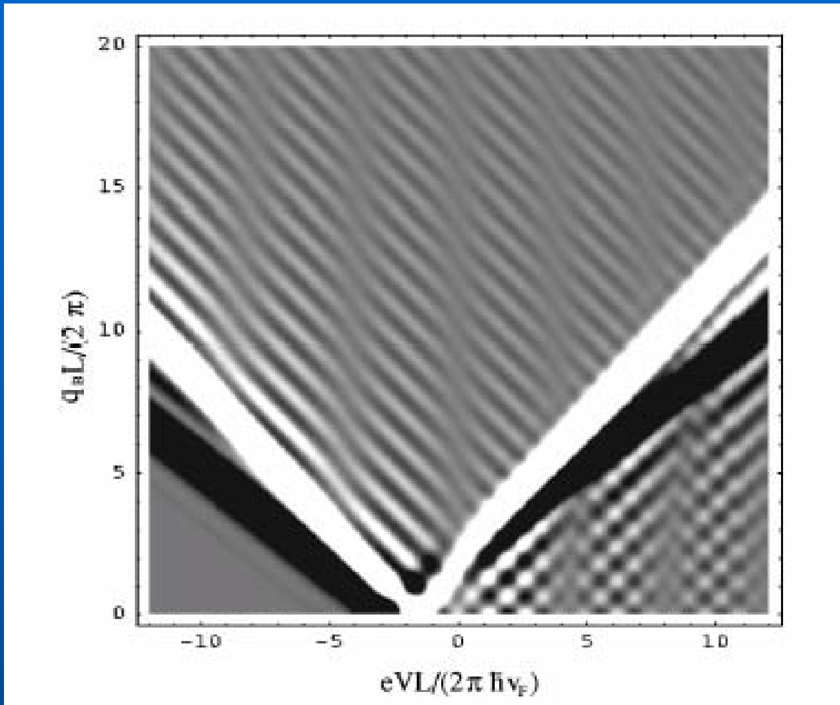


- Measures the band (velocities)
- Finite size wire:



Interferences

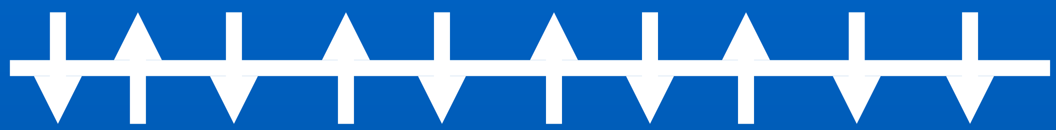




Interferences depend on u.

Instabilities

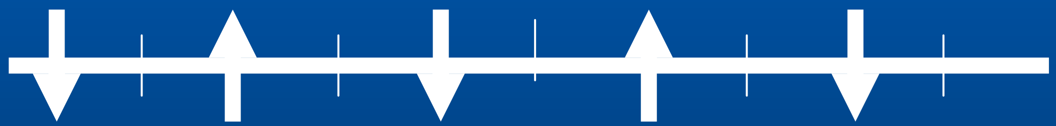
Mott insulator (commensurate)

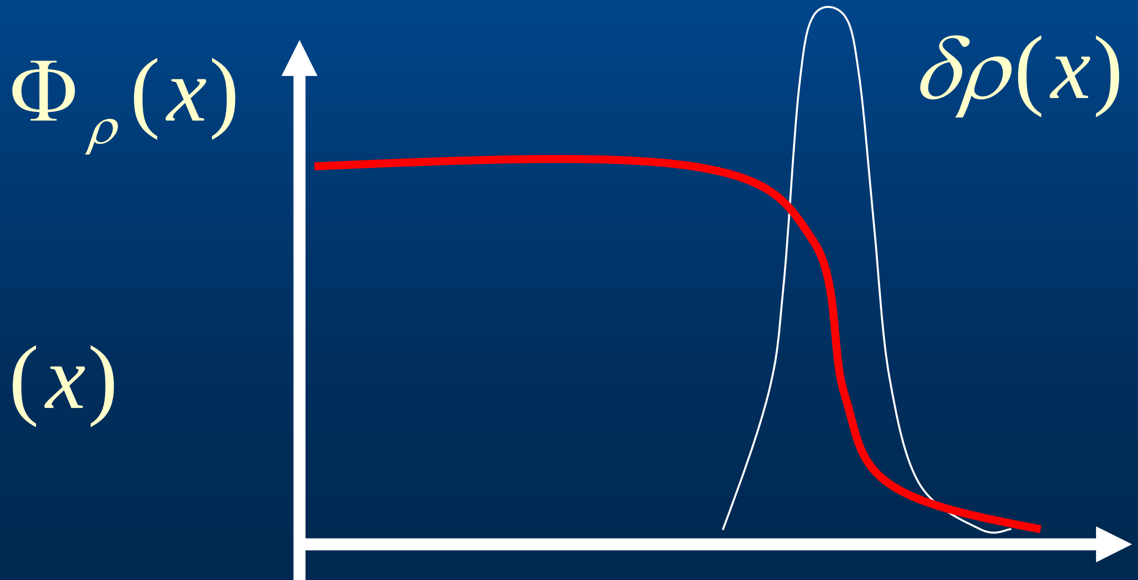


½ filling

U vs. t

Any commensurate filling works



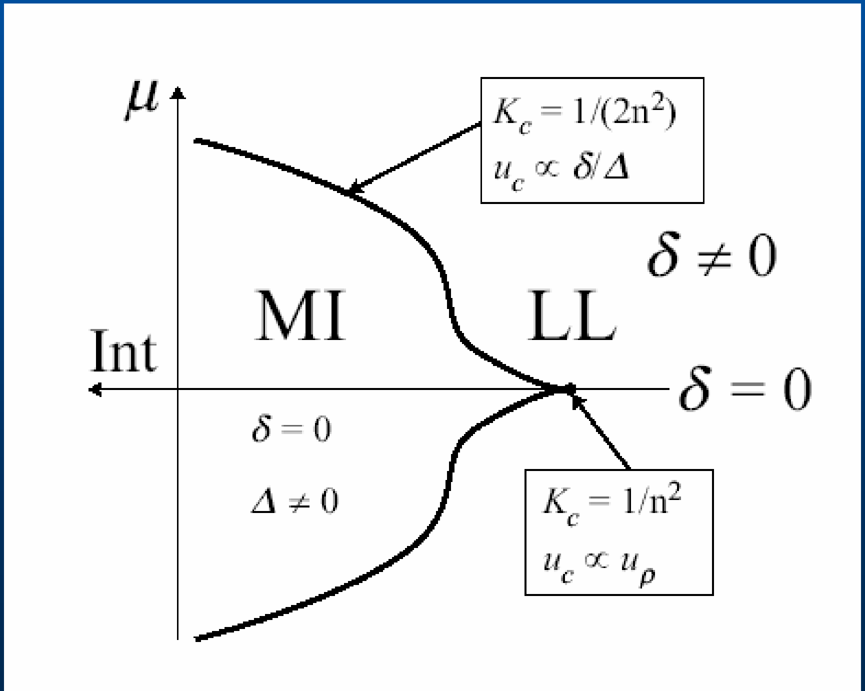


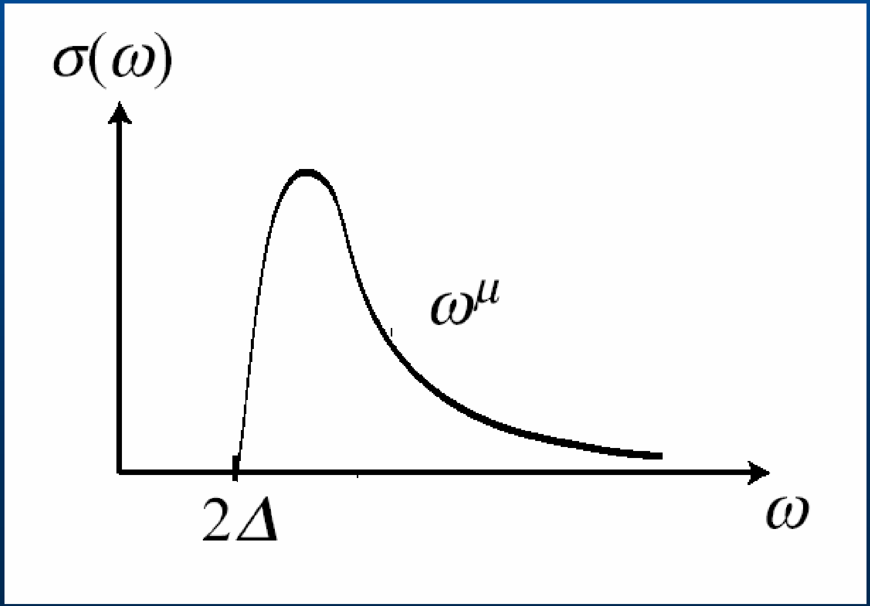
$$\delta\rho(x) = -\rho_0 \nabla \Phi_\rho(x)$$

Charges = Solitons in Φ(x)

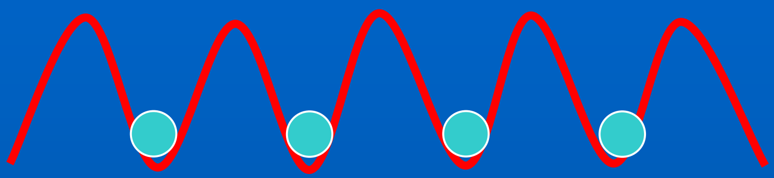
$$H = \int \frac{dx}{2\pi}[u_\rho K_\rho(\pi\Pi_\rho(x))^2 + \frac{u_\rho}{K_\rho}(\nabla\Phi_\rho(x))^2]$$

$$+ g(U)\int dx \cos(n\sqrt{8}\Phi_\rho) - \mu\int dx \nabla\Phi_\rho(x)$$

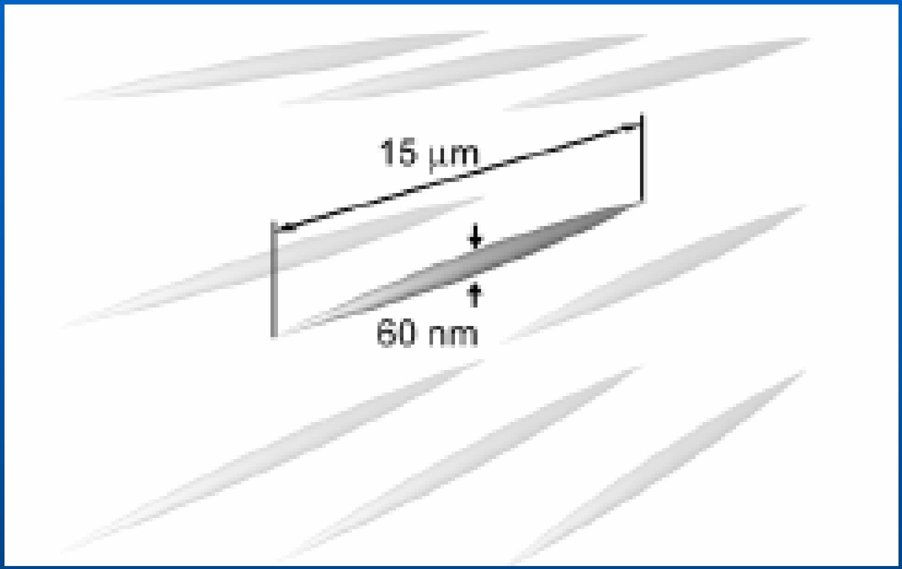




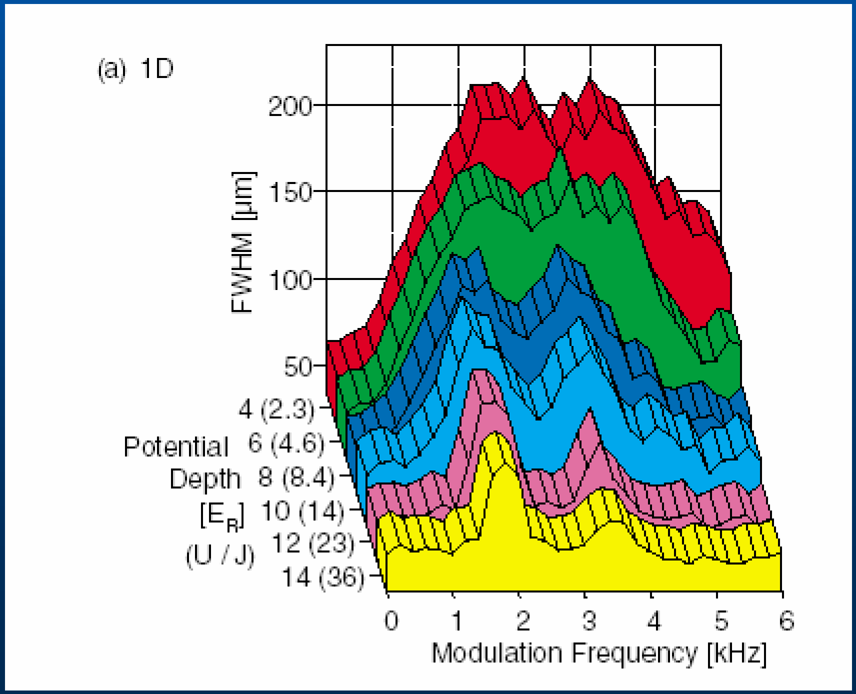
Mott transition in optical lattices



Mott insulator

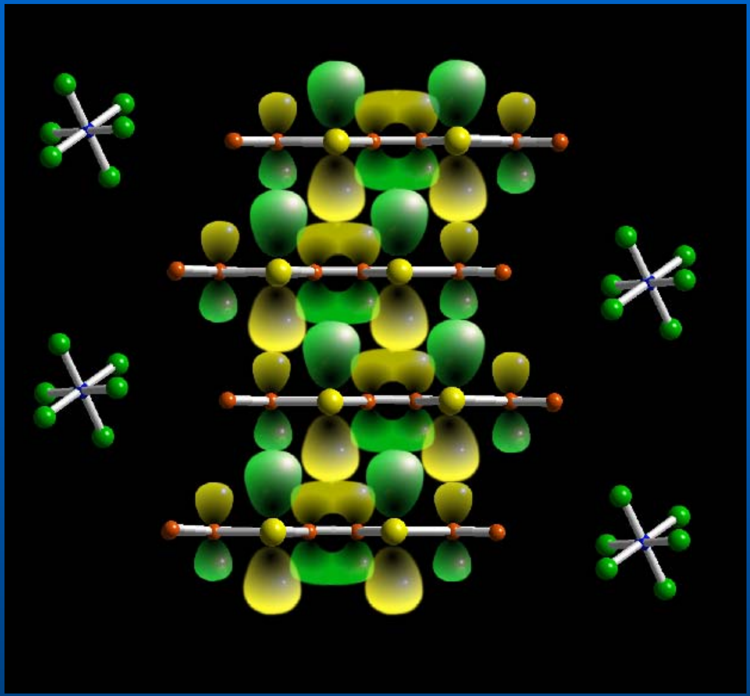


T. Stoferle *et al.* PRL **92** 130403 (2004)

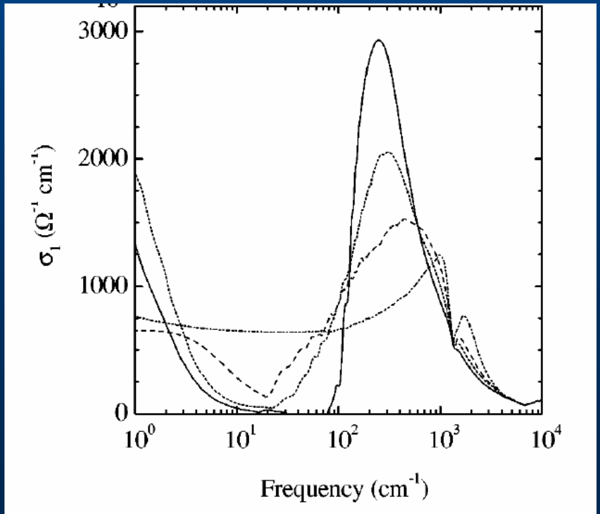


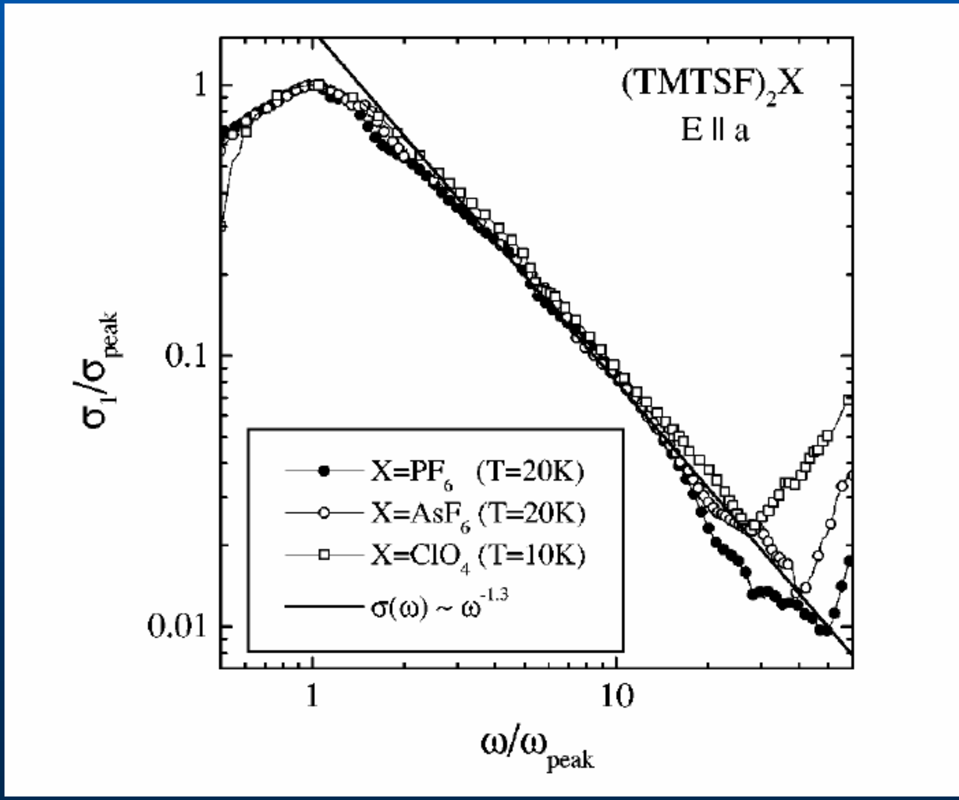
Bosons !

Optical conductivity



A. Schwartz et al. PRB 58 1261 (1998)





More than 1D

Anisotropic situation

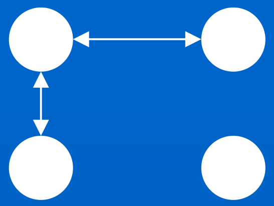
Dimensional Crossover Deconfinement

References :

- S. Biermann, A. Georges, T. Giamarchi and A. Lichtenstein, in ``Strongly Correlated Fermions and Bosons in Low Dimensional Disordered Systems'' (Kluwer), cond-mat/0201542.

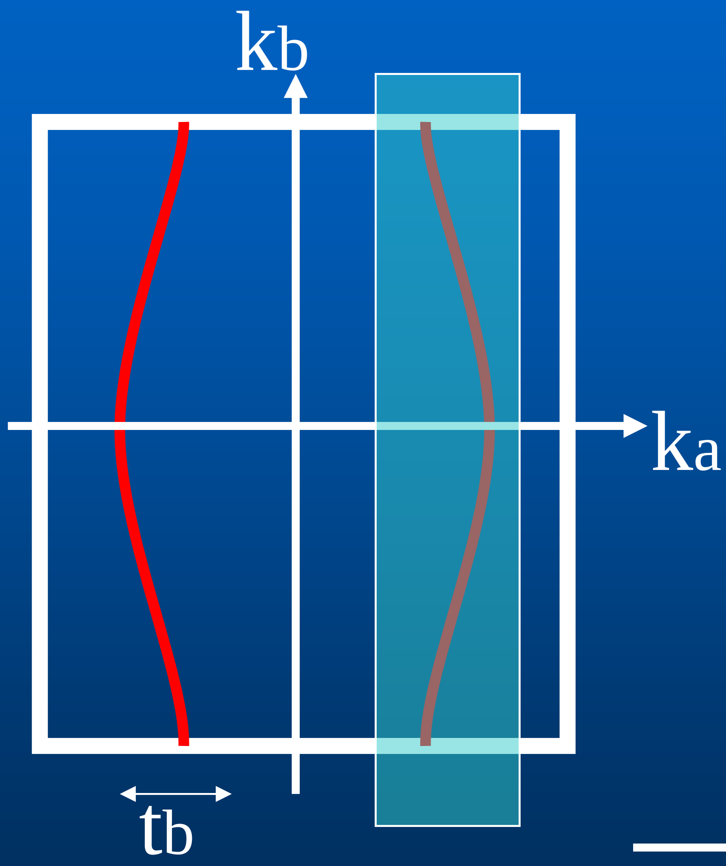
-T. Giamarchi, Chemical Review (2004)

And references therein.



$$t_a > t_b > t_c$$

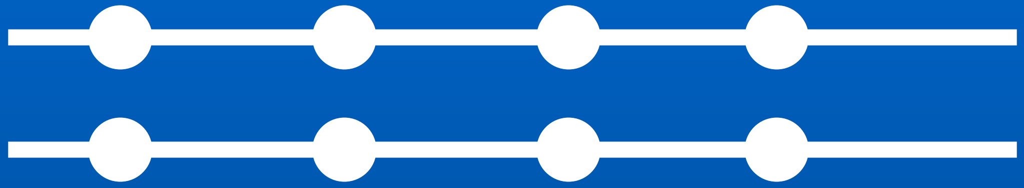
$$3000K, 300K, 20K$$



- High Energy (T,ω): 1D
- Low Energy (T,ω) : 2D,3D

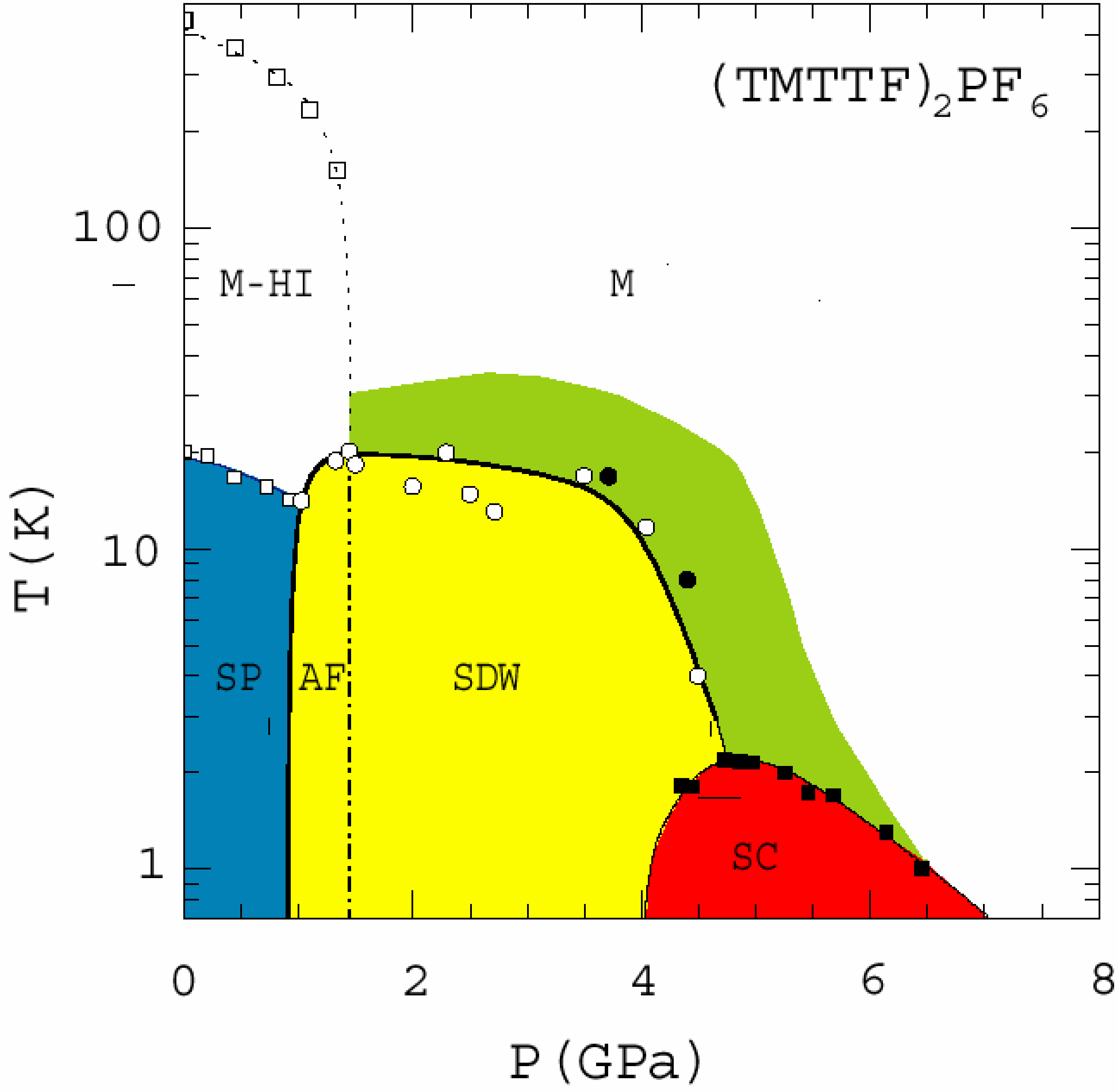
Dimensional crossover

Mott insulators: confinement



- 1 chain : Mott insulator U >0
- 3d : Mott insulator U > Uc

Competition Mott insulator/Interchain hopping



D. Jaccard et al., J. Phys. C, 13 L89 (2001)

Longitudinal vs Transverse

$T < t_{\|}$

Coherent transport

$$\sigma_{\|} \propto T^{3-4n^2K}$$

$$\sigma_{\|} \propto T^{-1.3}$$

$T > t_{\perp}$

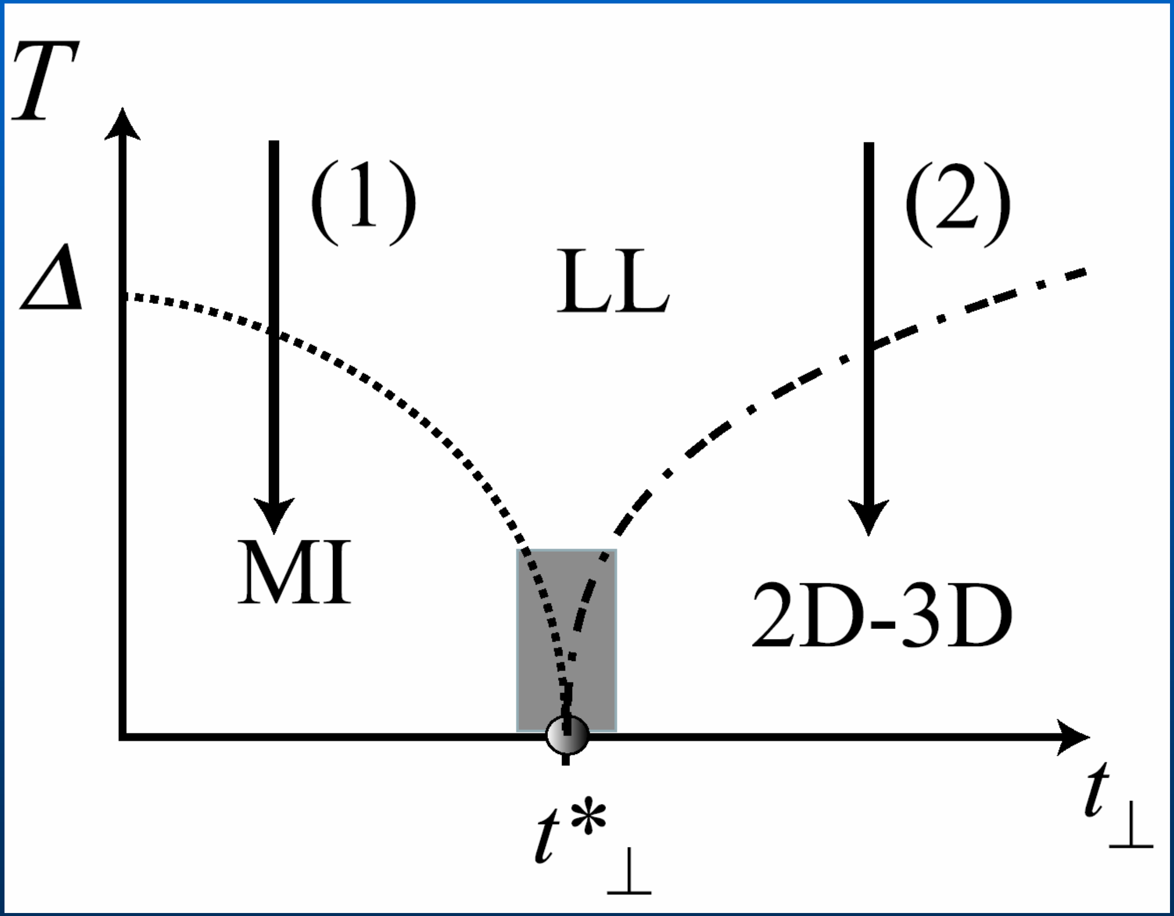
Tunnelling (SP density of states)

$$\sigma_{\perp}(\omega,T) \propto (\omega,T)^{2\alpha-1}$$

$$\alpha = \frac{1}{4}(K+K^{-1}) - \frac{1}{2}$$

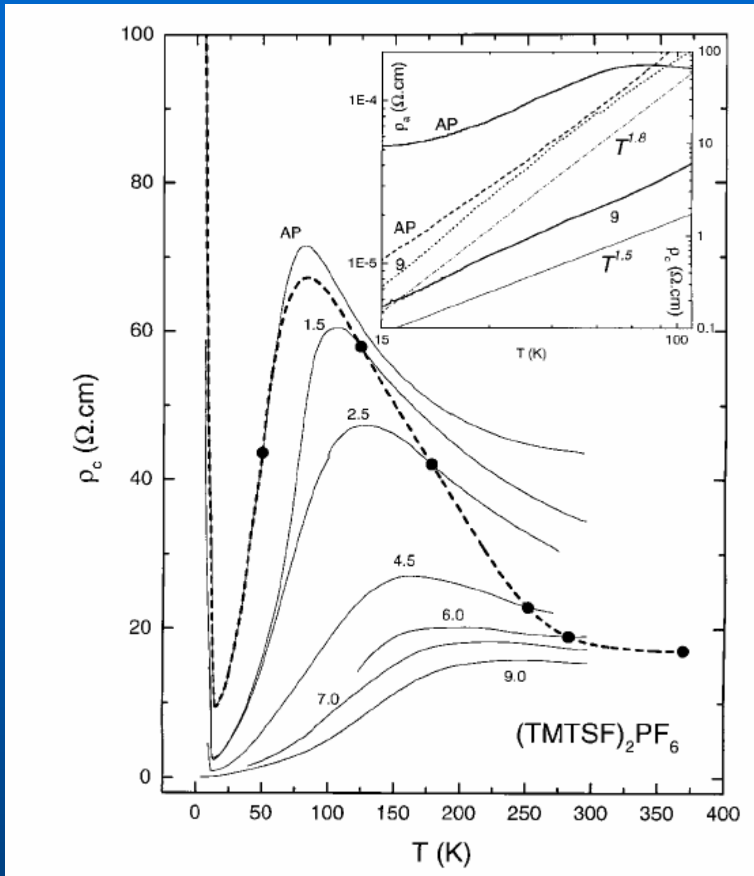
$\alpha \approx 0.6$ $\qquad K = 0.23$

Deconfinement

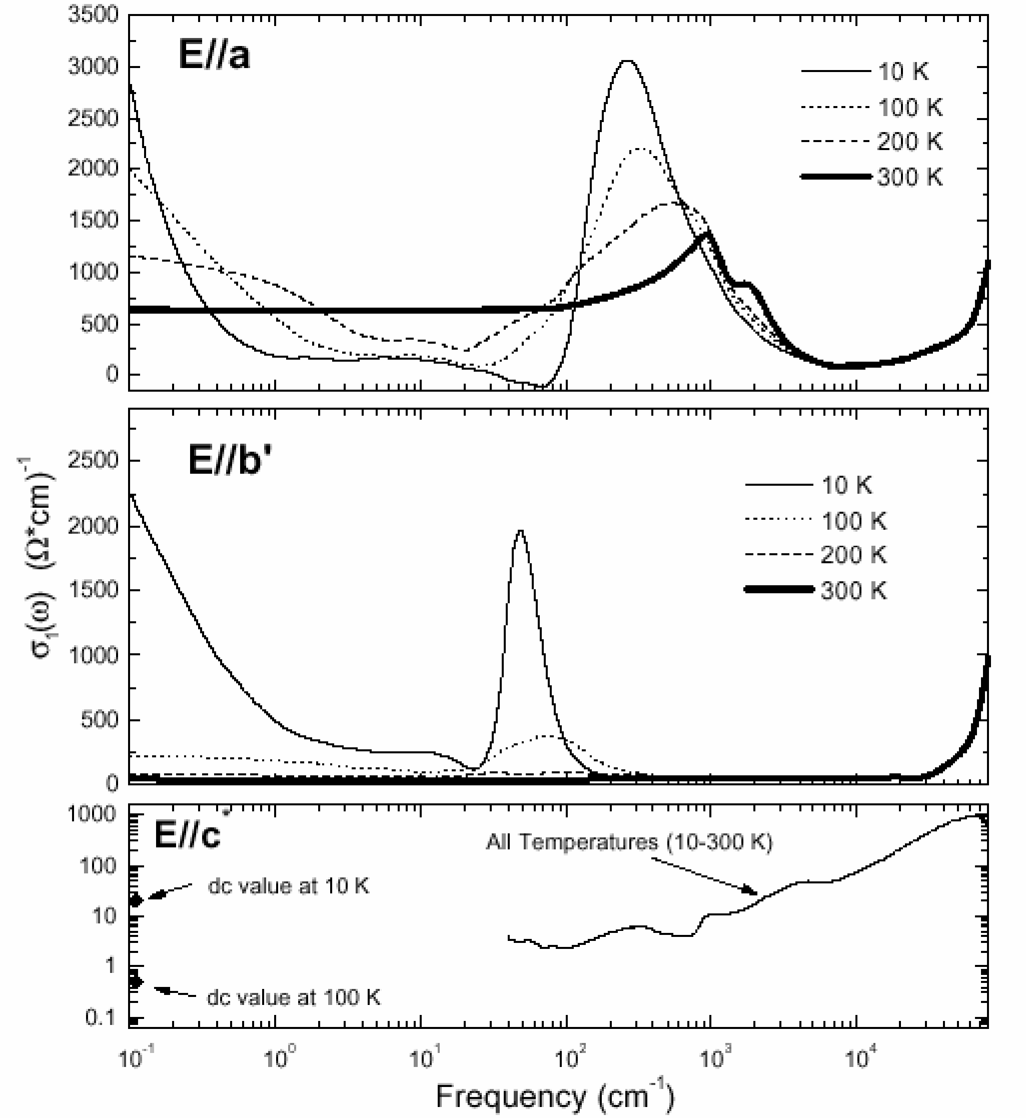


(1) LL-MI

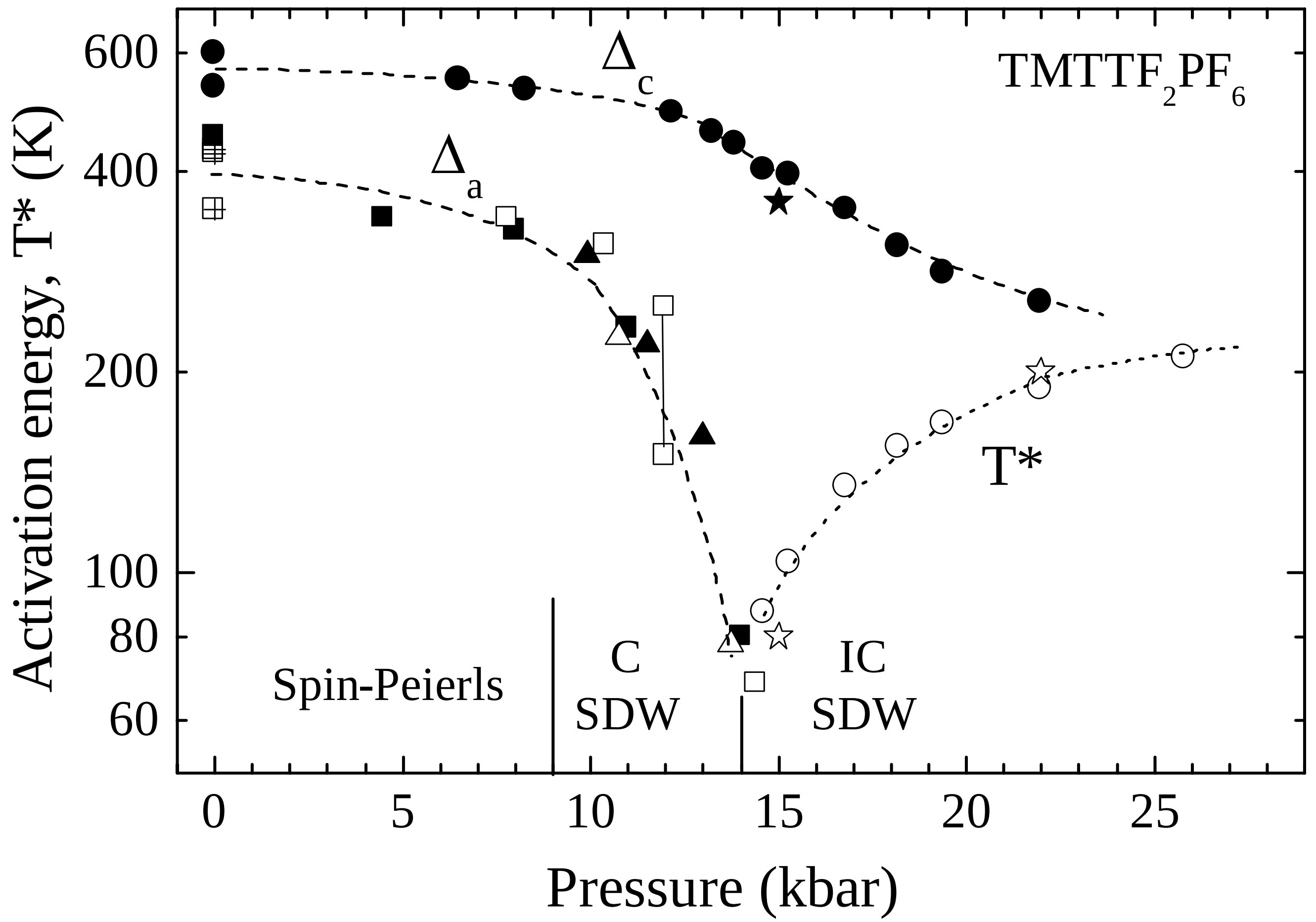
(2) Dimensional crossover



J. Moser et al. Euro Phys. J. B 1 39 (1998)



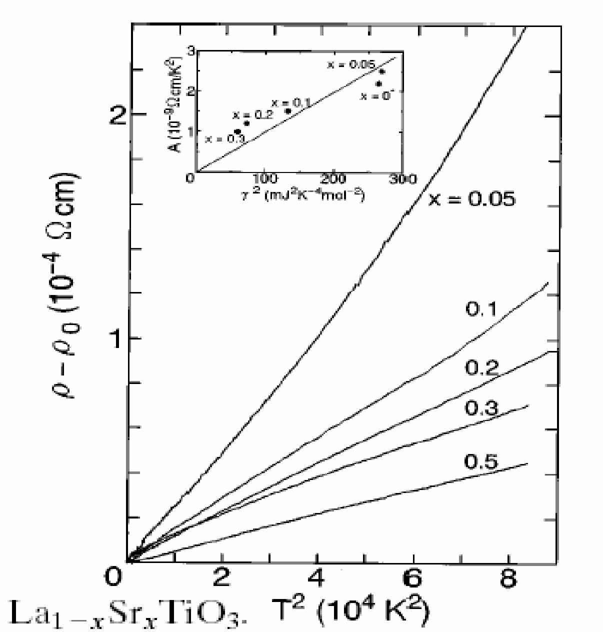
V. Vescoli et al. Euro Phys J B 11 365 (1999)



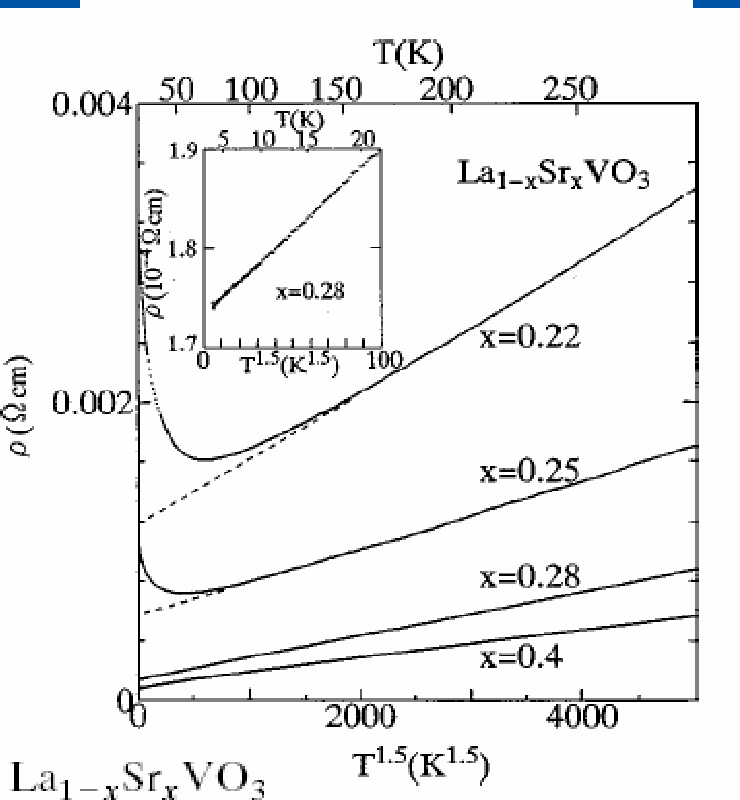
P. Auban-Senzier, D. Jérome, C. Carcel and J.M. Fabre J de Physique IV, (2004)

More than 1D

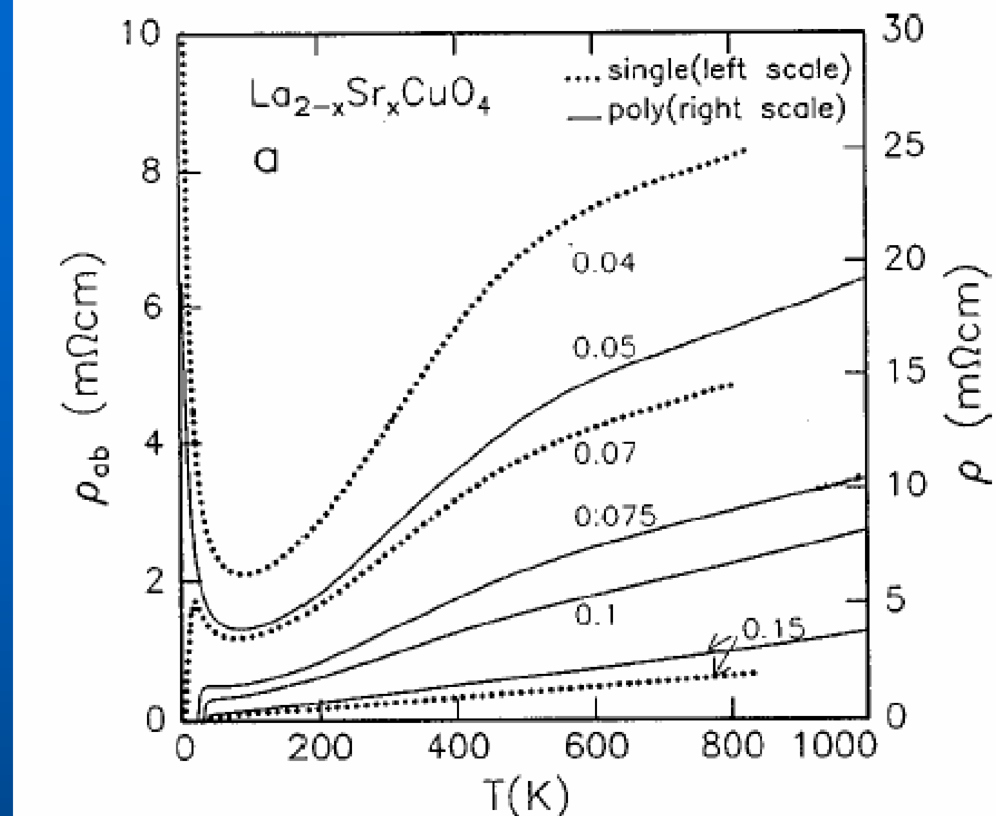
Isotropic situation



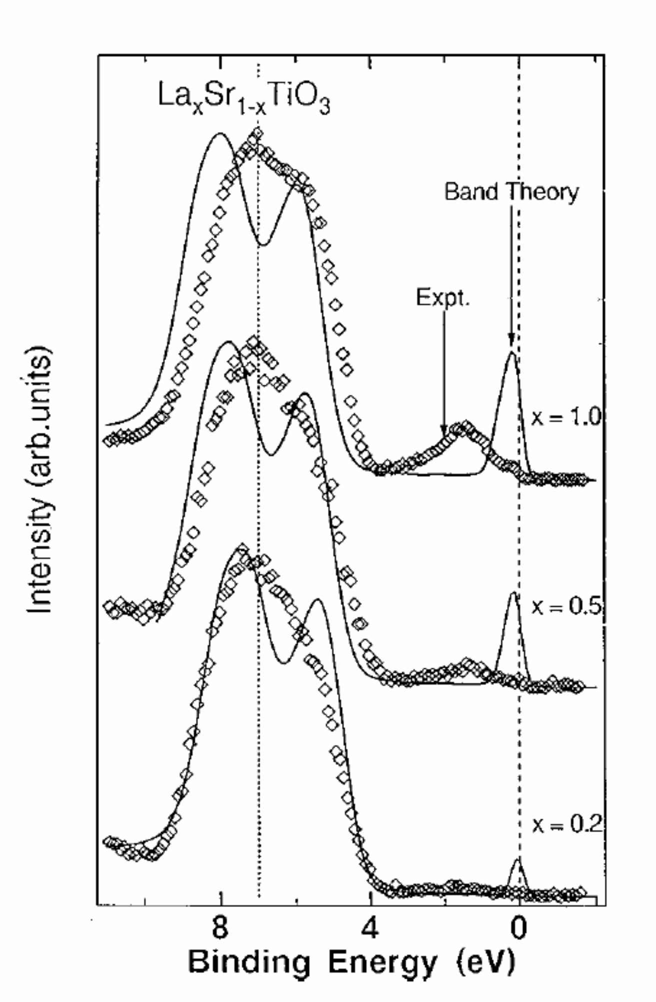
$\rho - \rho_0$ (10^{-4} Ωcm)
x = 0.05
0.1
0.2
0.3
0.5
$La_{1-x}Sr_xTiO_3$.
T^2 (10^4 K^2)



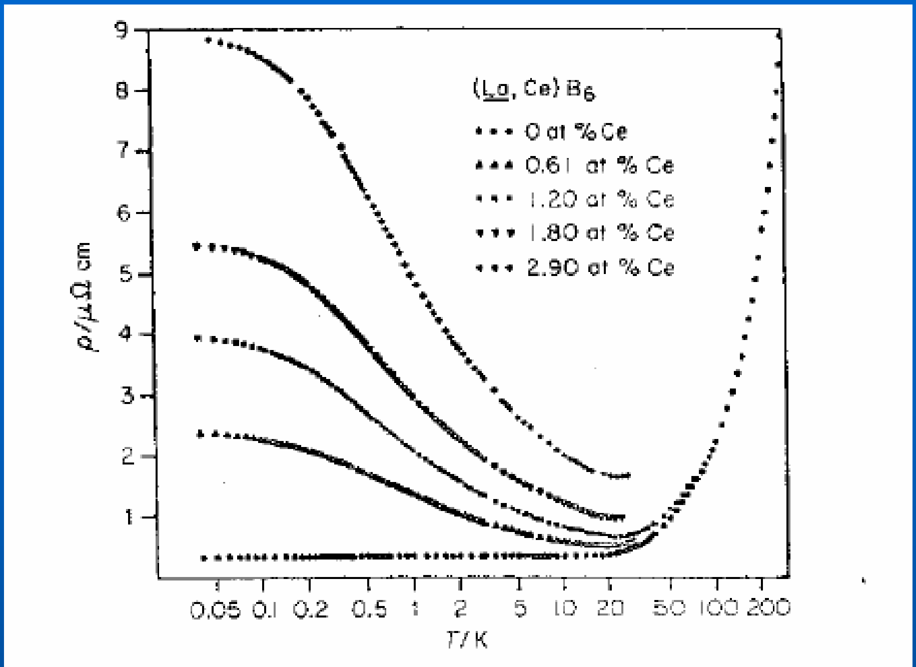
T(K)
$La_{1-x}Sr_xVO_3$
x=0.22
x=0.25
x=0.28
x=0.4
x=0.28
ρ (Ωcm)
$T^{1.5}$($K^{1.5}$)
$La_{1-x}Sr_xVO_3$



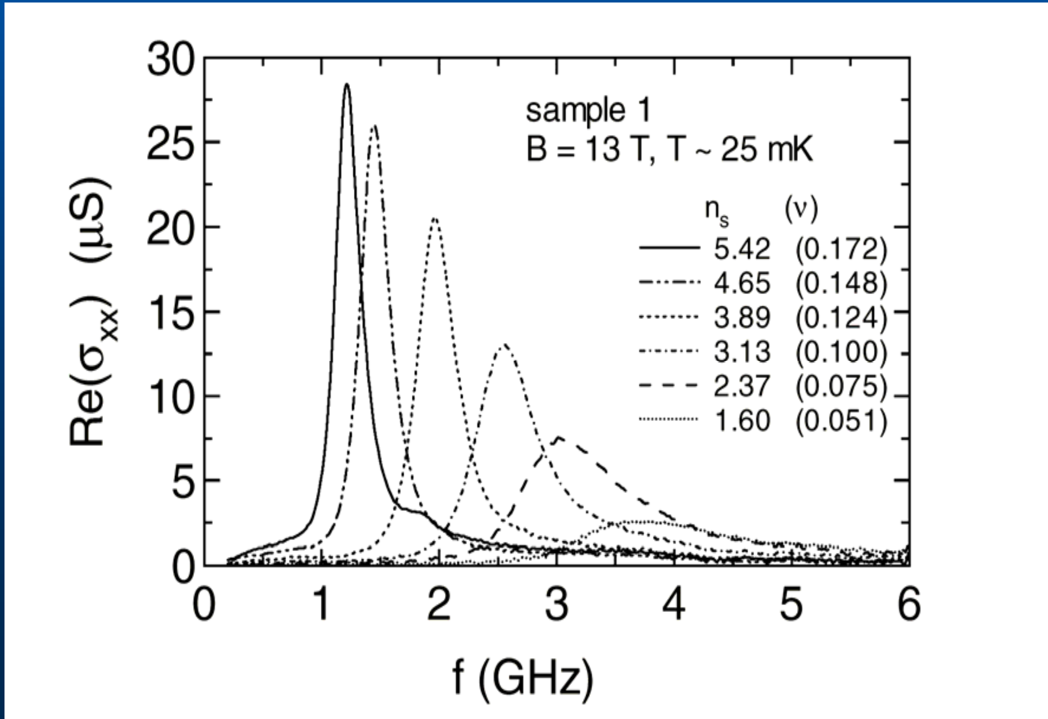
$La_{2-x}Sr_xCuO_4$
a
.... single(left scale)
— poly(right scale)
0.04
0.05
0.07
0:075
0.1
0.15
ρ_{ab} (mΩcm)
ρ (mΩcm)
T(K)



$La_xSr_{1-x}TiO_3$
Band Theory
Expt.
x = 1.0
x = 0.5
x = 0.2
Intensity (arb.units)
8
4
0
Binding Energy (eV)



(La, Ce) B_6
0 at % Ce
0.61 at % Ce
1.20 at % Ce
1.80 at % Ce
2.90 at % Ce
$\rho/\mu\Omega$ cm
T/K
0.05 0.1 0.2 0.5 1 2 5 10 20 50 100 200



sample 1
B = 13 T, T ~ 25 mK
n_s (ν)
5.42 (0.172)
4.65 (0.148)
3.89 (0.124)
3.13 (0.100)
2.37 (0.075)
1.60 (0.051)
$Re(\sigma_{xx})$ (μS)
f (GHz)

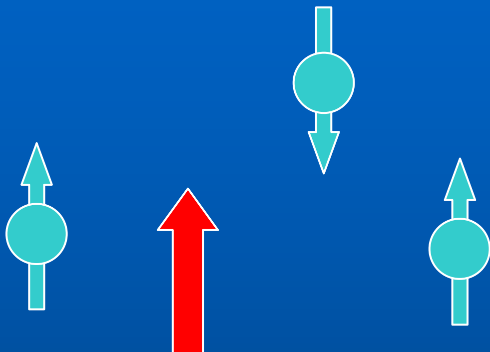
Non Fermi liquids

- Fake non Fermi liquids

Apparent NFL

- True NFL

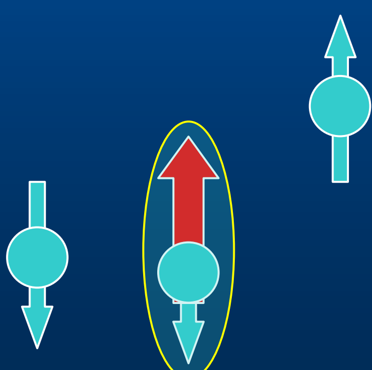
Two Channel Kondo problem



Kondo:

Spin + 3D Fermi liquid

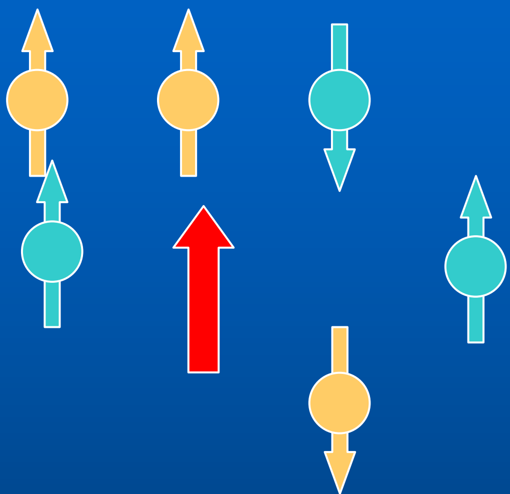
$$H = H_0 + J\vec{S}\cdot(\psi^\dagger(0)\mathcal{S}\psi(0))$$



Total screening:
Fermi liquid

2Channels Kondo:

Spin + 2 × 3D Fermi liquid



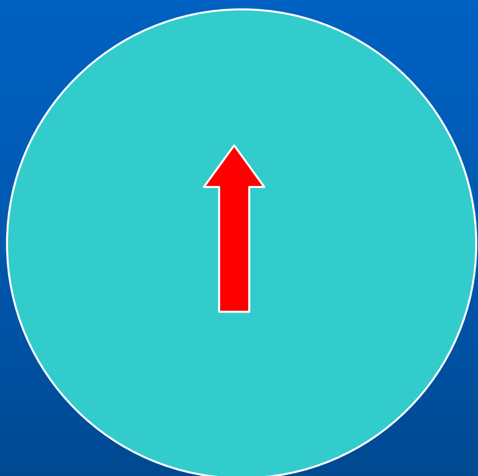
$$H = H_0 + J\vec{S}\cdot\left[\psi_1^\dagger(0)\mathcal{S}\psi_1(0) + \psi_2^\dagger(0)\mathcal{S}\psi_2(0)\right]$$

Over screening:Non Fermi liquid

$$\chi^{\mathrm{imp}}(\omega,T)\sim\frac{1}{T_*}\ln\left[\frac{T_*}{\max(\omega,T)}\right]\ ;\quad(\omega,T\ll T_*)\ ,$$

$$C^{\mathrm{imp}}(T)\sim\frac{T}{T_*}\ln\frac{T_*}{T}\ ;\quad(T\ll T_*)$$

In fact ....



One dimensional problem

Half space only

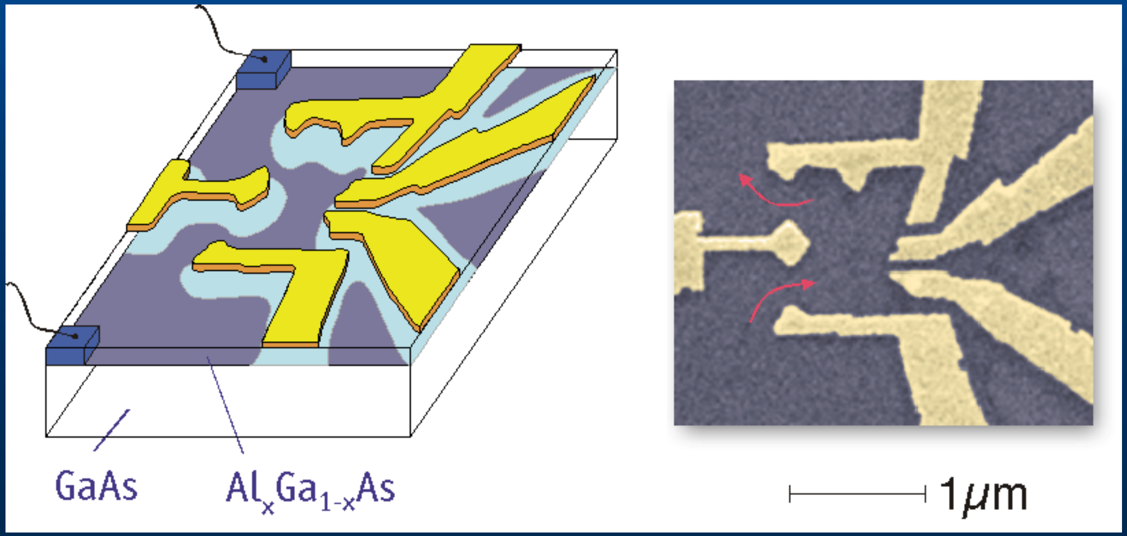
Boundary problem: conformal invariance

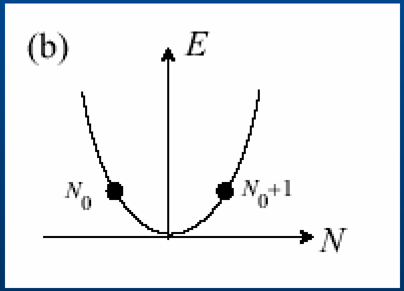
How to check

- Heavy fermions (f electrons)

Many impurities ?

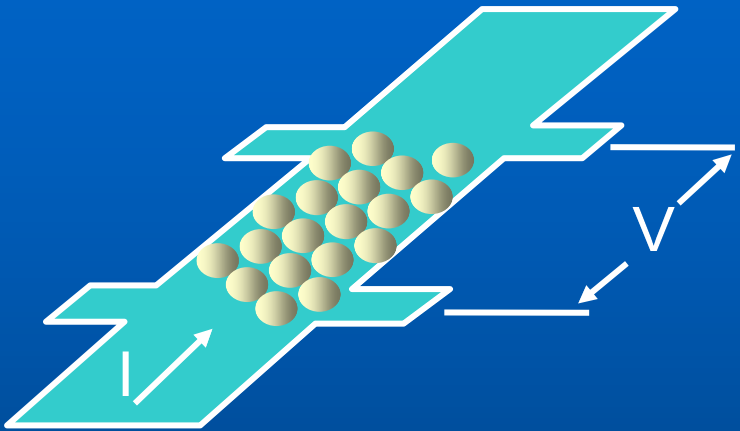
- Quantum dots





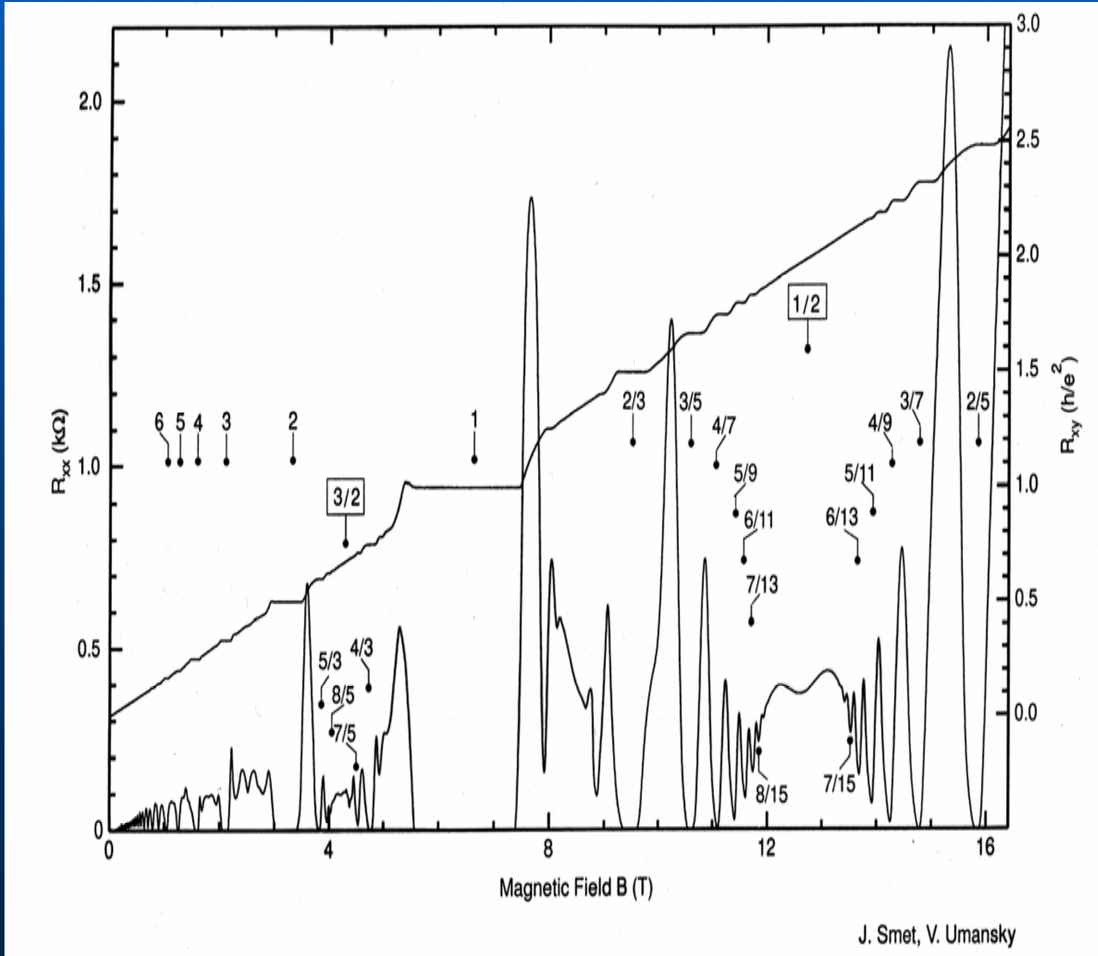
Difficult

Fractional quantum Hall effect



Incompressible

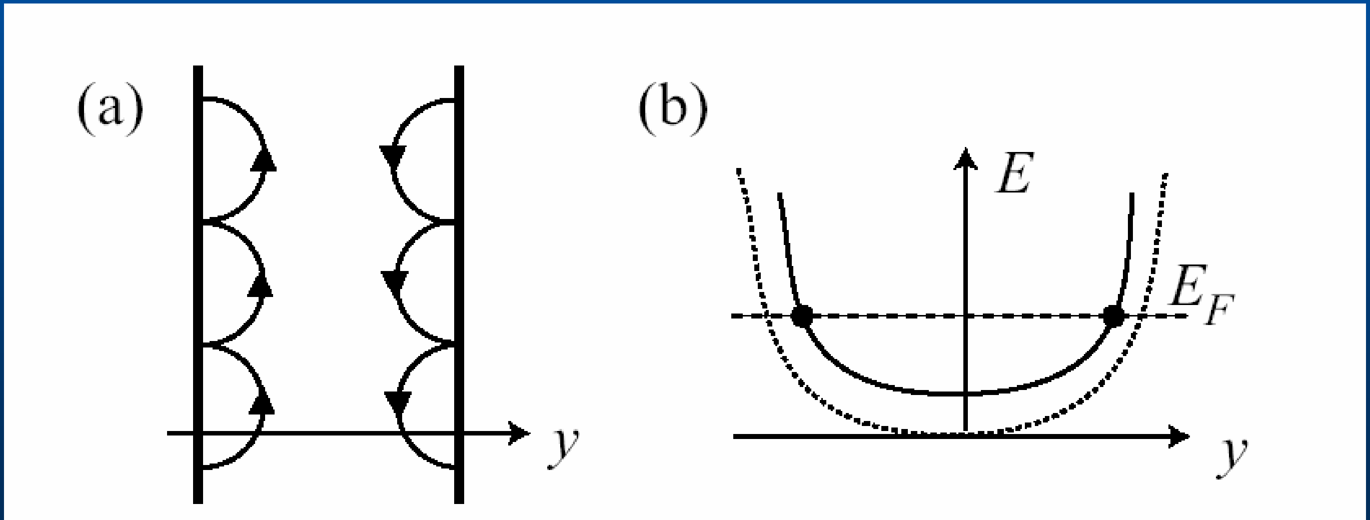
Liquid



- Fractional charges (Laughlin QP)

electron $\rightarrow 3 \times (e/3)$

- But ....



Chiral Luttinger liquid

References

- Kondo/Multichannel Kondo:

D. L. Cox and A. Zawadowski Adv. Phys, 47 599 (1998)

Edge states:

X. G. Wen Adv. Phys. 44 405 (1995)

M.P.A Fisher and L. Glazman in ``Mesoscopic Electron Transport'' (Kluwer); cond-mat/9610037

For connections with Luttinger liquids see also:

T. Giamarchi ``Quantum Physics in one dimension'' (Oxford).

Marginal FL

- High Tc $\rho(T) \sim T$ $\sigma(\omega) \sim 1/\omega$

$$\tau^{-1} = Max(\omega, T)$$

- How to get and keep a Fermi surface

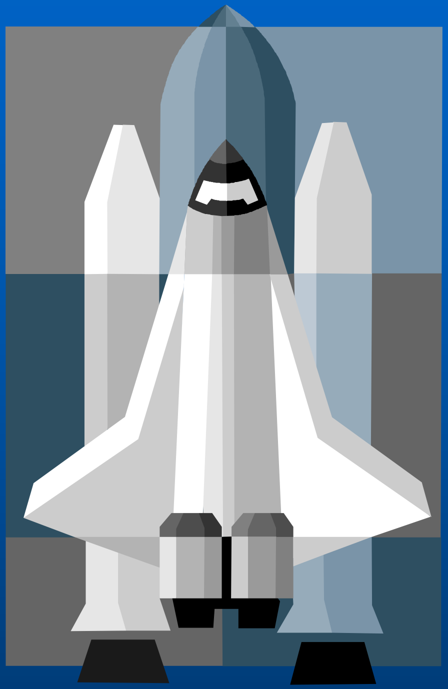
(C. M. Varma, P. B. Littlewood, S. Schmitt-Rink, E. Abrahams and A. E. Ruckenstein 63 1996 (1989))

$$\left\langle \psi\ (\omega, q)\psi^*(\omega, q)\right\rangle = \frac{1}{\omega - \varepsilon(k) - \Sigma(\omega, k)}$$

$$FL:\ \ \mathrm{Re}\,\Sigma \sim \omega\ ;\ \mathrm{Im}\,\Sigma \sim \omega^2$$

$$MFL:\ \ \mathrm{Re}\,\Sigma \sim \omega \log(x)\ ;\ \mathrm{Im}\,\Sigma \sim x \qquad x = \max(\omega, T)$$

Beyond 1d



To boldly go where no fermion has gone before !!

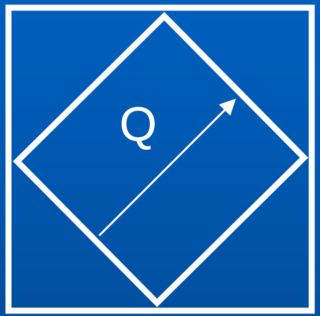
$$\langle \psi_R(x)\psi_R^*(0)\rangle = \frac{Z_k}{\omega - E(k) - i\Gamma} + G_{inc}$$

- No quasiparticles: $Z_k \propto \log(\Lambda / \varepsilon_k)$

How to get ?

Close to an instability

- ``Nested Fermi Liquid''



$$\chi_{\mathrm{NFL}}(Q) = \frac{\pi N(0)}{2} \tanh(\omega/4T)$$

- ``Nearly Antiferromagnetic Fermi Liquid''

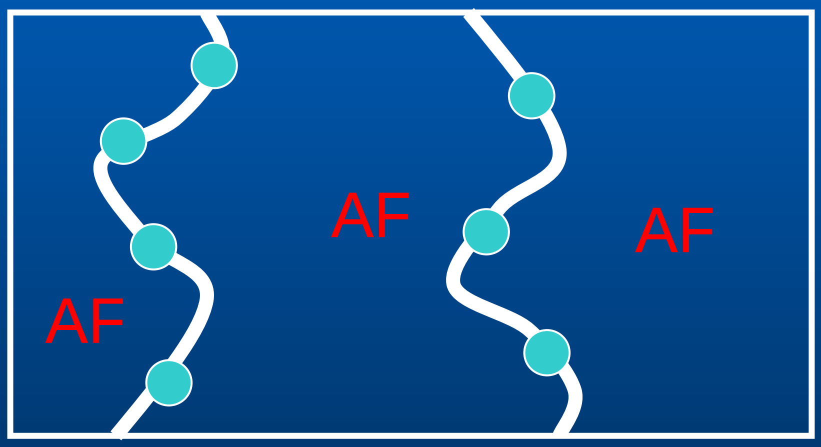
$$\chi_{\mathrm{NFL}}(Q) = \frac{\chi_q}{1 + \xi^2 (Q-q)^2 - i\omega/\omega_0}$$

(A. J. Millis and H. Monien and D. Pines, PRB 47 167 (1990))

But: close to an instability !

Stripes

- 1D in two dimensions !

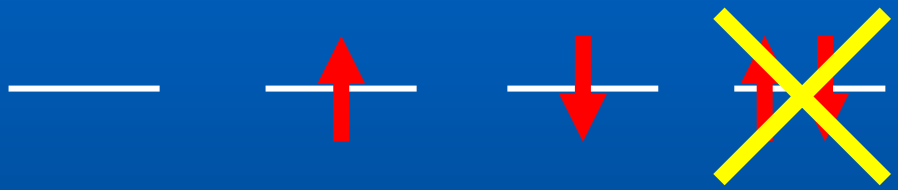


Phase separation

- Properties of a Luttinger liquid

S. A. Kivelson and E. Fradkin and V. J. Emery, Nature 393 550 (1998)

Strong interactions



U large

$$c_\uparrow^\dagger = f_\uparrow^\dagger b$$

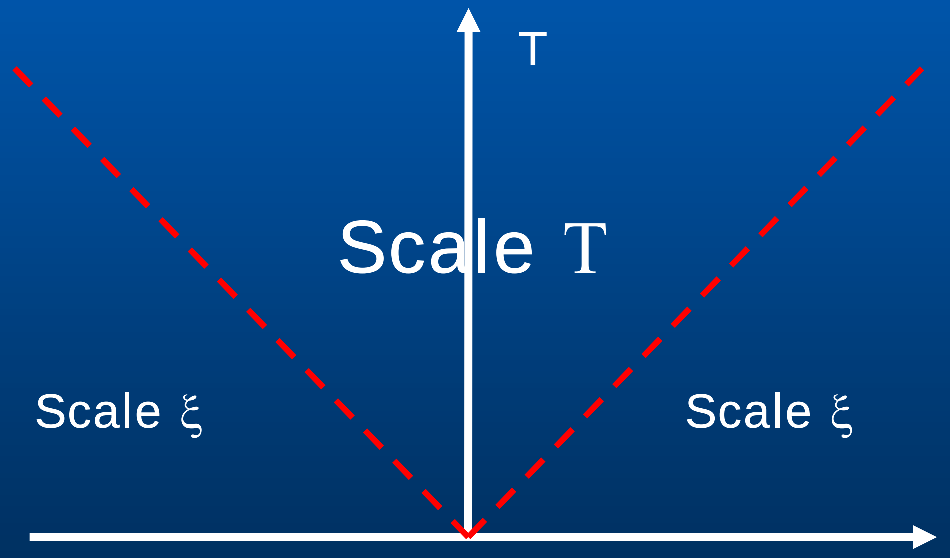
Spinon Holon

$$n_\uparrow + n_\downarrow + n_b = 1$$

But: constraint

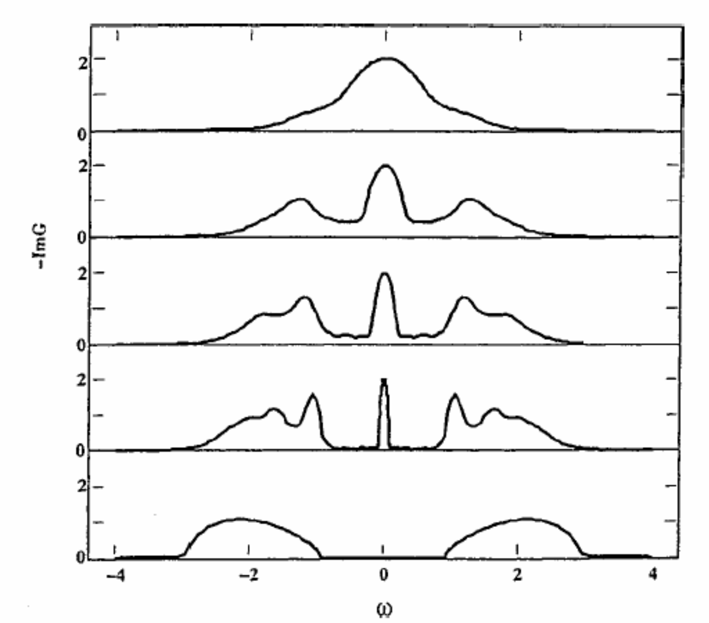
Quantum critical point

- Idea: critical point means $\xi = \infty$



S. Sachdev ``Quantum Phase Transitions'' (Cambridge)

Mott transition



Incoherent regime

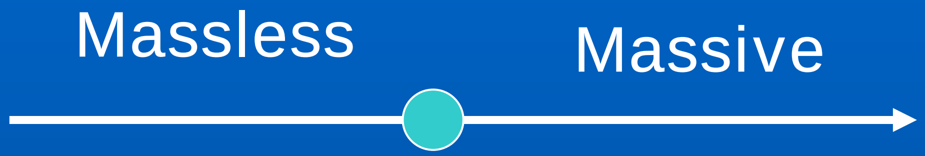
$$T_{\text{coh}} = \delta^{z/d}$$

Mean Field: z=2

Hubbard d=2: z=4 ?

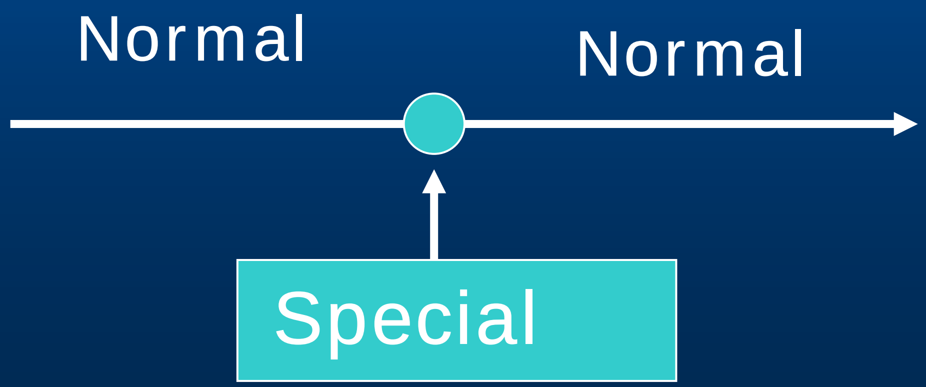
M. Imada and A. Fujimori and Y. Tokura, Rev. Mod. Phys. 70 1039 (1998).

- Usually: QCP



QCP reflects one of the phases

- Try to find QCP:



Additional references

C. M. Varma, Z. Nussinov and W. van Saarloos, Phys. Rep. 361 267 (2002).

P.W. Anderson ``The Theory of Superconductivity in the High-Tc Cuprate Superconductors'' (Princeton University Press).

Conclusions

Two well established electronic liquids

- « d=3 » : Fermi liquid; Excitations are « like » free fermions
- d=1 (and extensions)
  Luttinger liquid; Only collective excitations; non universal exponents

Other types of non fermi liquids (d=2) ???????????