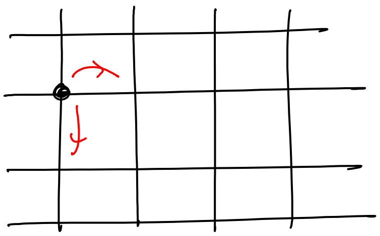
Basic Models

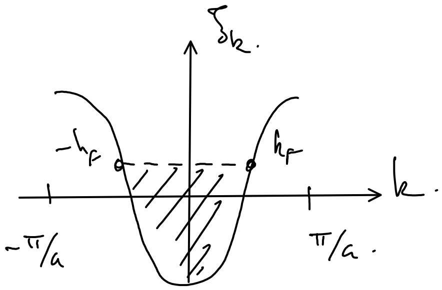
I) Hubbard Model :



fermions

$$H_{Kin} = -t \sum_{\substack{\langle ij \rangle \\ \sigma}} \left(c^{+}_{i\sigma} c_{j\sigma} + c^{+}_{j\sigma} c_{i\sigma} \right)$$

(tight binding)



$$\xi_k = -2t\cos(k_x) - 2t\cos k_y \dots$$

$$H_{int} = U \sum_i n_{i\uparrow} n_{i\downarrow}$$

$$H = H_{Kin} + H_{int}$$

$$\boxed{\begin{aligned} H = & -t \sum_{\substack{\langle ij \rangle \\ \sigma}} \left(c^{+}_{i\sigma} c_{j\sigma} + h.c \right) + U \sum_i \left(n_{i\uparrow} - \tfrac{1}{2}\right)\left(n_{i\downarrow} - \tfrac{1}{2}\right) \\ & - \mu \sum_i \left(n_{i\uparrow} + n_{i\downarrow} - 1 \right) - \frac{h}{2} \sum_i \left(n_{i\uparrow} - n_{i\downarrow} \right) \end{aligned}}$$

---	$\uparrow$	$\downarrow$	$\uparrow\downarrow$
0	0	0	U
$\frac{U}{4}$	$-\frac{U}{4}$	$-\frac{U}{4}$	$\frac{U}{4}$

$$\langle n_{i\uparrow} + n_{i\downarrow} \rangle = 1 \longrightarrow \mu = 0 \quad \forall T$$

Particle hole Symmetry

bipartite lattice

$$\begin{matrix} A \rightarrow B & A & B & A \\ \downarrow & & & \\ B & A & B & A & A \end{matrix}$$

$$c^{+}_{i\uparrow} \rightarrow (-1)^i d_{i\uparrow}$$

$$H_{Kin} = -t \sum_{\langle ij \rangle} \underbrace{(-1)^i (-1)^j}\ \underbrace{d_{i\sigma} d^{+}_{j\sigma}} + h.c.$$

$\underbrace{\qquad}_{(-1)} \quad \underbrace{\qquad}_{-d^\dagger_{j\sigma} d_{i\sigma}}$

$c^\dagger_{i\downarrow} \to (-1)^i d_{i\downarrow}$

$H_{kin}[c] \to H_{kin}[d]$

$\sum_i (n_{i\uparrow} - 1/2)(n_{i\downarrow} - 1/2)$

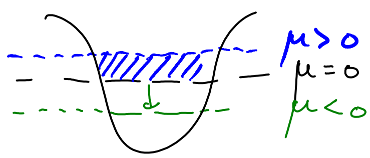
$c^\dagger_{i\sigma} c_{i\sigma} \to d_{i\sigma} d^\dagger_{i\sigma} \to 1 - d^\dagger_{i\sigma} d_{i\sigma}$

$n_{i\uparrow} - 1/2 \to 1/2 - d^\dagger_{i\sigma} d_{i\sigma}$

$\sum_i (c^\dagger_{i\uparrow} c_{i\uparrow} - 1/2)(c^\dagger_{i\downarrow} c_{i\downarrow} - 1/2) \to \sum_i (d^\dagger_{i\uparrow} d_{i\uparrow} - 1/2)(\quad \downarrow \quad)$

$-\mu \sum_i (n_{i\uparrow} + n_{i\downarrow} - 1) \to -\mu \sum_i (1 - n_{i\uparrow} - n_{i\downarrow})$

$\langle c^\dagger_{i\sigma} c_{i\sigma} \rangle \to 1 - \langle d^\dagger_{i\sigma} d_{i\sigma} \rangle$



$\langle n \rangle = 1$

\# $\begin{cases} c^\dagger_{i\uparrow} \to d^\dagger_{i\uparrow} \\ c^\dagger_{i\downarrow} \to (-1)^i d_{i\downarrow} \end{cases}$?

$n_{i\downarrow} \to 1 - n_{i\downarrow}$ $\qquad (n_{i\uparrow} - 1/2)(n_{i\downarrow} - 1/2) \to -(\quad)(\quad)$

$U \to -U$

$n_{i\uparrow} + n_{i\downarrow} - 1 \to n_{i\uparrow} - n_{i\downarrow}$

$n_{i\uparrow} - n_{i\downarrow} \to n_{i\uparrow} + n_{i\downarrow} - 1$

c	d
U	$\to -U$
μ	$\to h$
h	$\to \mu$

$\langle n_{i\uparrow} + n_{i\downarrow} \rangle = 1$

z component $\langle c^\dagger_{i\uparrow} c_{i\uparrow} - c^\dagger_{i\downarrow} c_{i\downarrow} \rangle \to d^\dagger_{i\uparrow} d_{i\uparrow} - d_{i\downarrow} d^\dagger_{i\downarrow}$

$= d^\dagger_{i\uparrow} d_{i\uparrow} + d^\dagger_{i\downarrow} d_{i\downarrow} - 1$

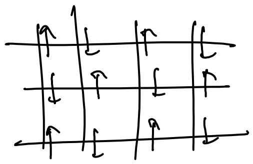
$\uparrow \; \downarrow \; \uparrow \; \downarrow \; \uparrow \; \downarrow \; \uparrow \quad \Rightarrow \quad \uparrow\downarrow \; \circ \; \uparrow\downarrow \; \circ \; \uparrow\downarrow \; \circ$

$SDW_z \longleftrightarrow CDW$

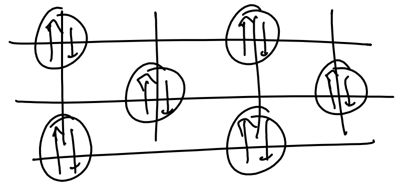
$SDW_x \qquad c^\dagger_{i\uparrow} c_{i\downarrow} + h.c.$

$(-1)^i \, d^{+}_{i\uparrow} d^{+}_{i\downarrow} + h.c. \rightarrow$ SU order parameter

SDW_y $\rightarrow$ SU order parameter



$\Rightarrow$



$n = 1$

PHYSICAL REVIEW A **79**, 033620 (2009)

Quantum simulation of the Hubbard model: The attractive route

A. F. Ho,[1] M. A. Cazalilla,[2,3] and T. Giamarchi[4]

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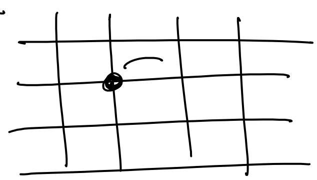
10. arXiv:0712.1808 [pdf, other]
The FFLO state in the one-dimensional attractive Hubbard model and its fingerprint in the spatial noise correlations
Andreas Luscher, Reinhard M. Noack, Andreas Laeuchli
Comments: 8 pages, 4 figures
Journal-ref: Phys. Rev. A 78, 013637 (2008)
Subjects: **Strongly Correlated Electrons (cond-mat.str-el)**

bosons



$$H = -t \sum_{\langle ij \rangle} \left(b^{+}_i b_j + h.c. \right) + U \sum_i n_i (n_i - 1)$$

$n_i = 0 \rightarrow 0$

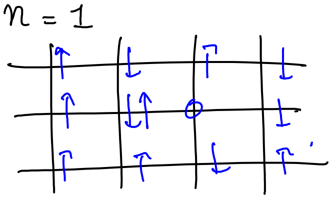
$n_i = 1 \rightarrow 0$

$n_i = 2 \rightarrow U$

[Bose-Hubbard model]

II) Basic Properties:

$n = 1$



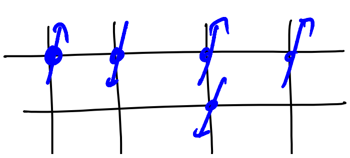
U small.

ground state $|\psi\rangle = \prod_{k < k_F} c^{+}_{k\sigma} |\phi\rangle$

$P \sim 1/4$

$\langle n_{i\uparrow} n_{i\downarrow} \rangle = \frac{1}{4}$

$U \sum_i n_{i\uparrow} n_{i\downarrow} \approx \frac{U}{4}$

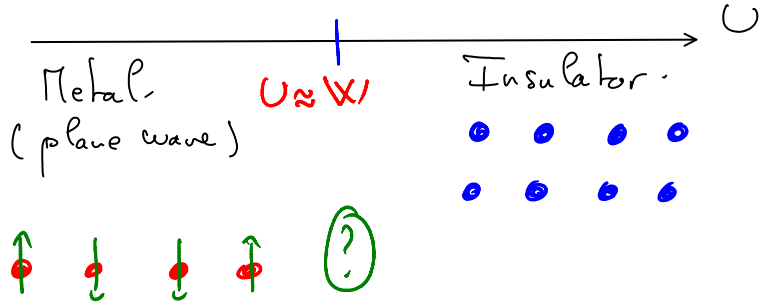


Localized ground state.

$\langle H_{kin} \rangle = 0$

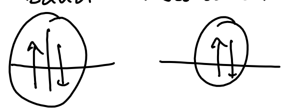
$$\langle H_U \rangle = 0$$

transition $\rightarrow$ Mott transition.



III.] Heisenberg Model:

band insulator

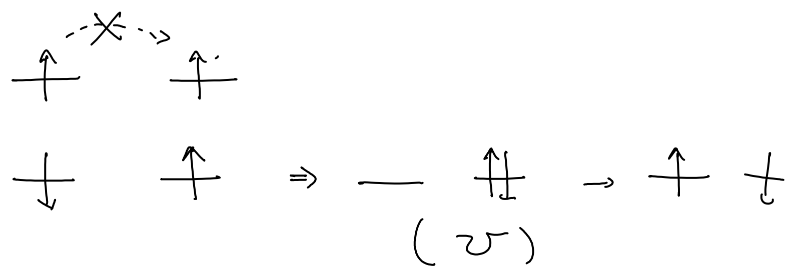


$S = 0$

Mott Insulator

$\uparrow$ or $\downarrow$

P.W. Anderson.



$$J \approx \frac{t^2}{U}$$

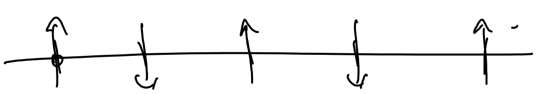
$$H = J \sum \vec{S}_i \cdot \vec{S}_j \qquad S_\alpha = \frac{1}{2}\sigma_\alpha$$

Antiferromagnetic exchange $\left(\frac{4t^2}{U}\right)$

Pauli Matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$[\sigma_x, \sigma_y] = \sigma_z \quad + \text{ permutations}$$



$$H = J \sum_{\langle ij \rangle} \left[S_x^i S_x^j + S_y^i S_y^j + S_z^i S_z^j \right]$$

$$\begin{cases} S^+ = S_x + i S_y \\ S^- = S_x - i S_y \end{cases} \qquad \begin{aligned} S^+ |\downarrow\rangle &= |\uparrow\rangle \\ S^+ |\uparrow\rangle &= 0 \end{aligned}$$

$$H = \left[\frac{J}{2} \sum_{\langle ij \rangle} \left[S_i^+ S_j^- + S_i^- S_j^+ \right] + J_z \, S_i^z S_j^z \right]$$

(Heisenberg Model) (Hubbards: $J_z = J_{xy}$)

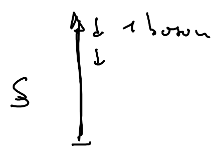
IV) Relations between the models:

$$H = \frac{J}{2} \sum_{\langle ij \rangle} \left(S_i^+ S_j^- + S_i^- S_j^+ \right) + J_z \sum_{\langle ij \rangle} S_i^z S_j^z$$

$$[S_x, S_y] = i S_z \quad \ldots \qquad [S_i, S_j] = 0$$

Holstein + Primakoff. (Phys. Rev. 58 1098 (1940))

$$\begin{cases} S^- = b^+ \sqrt{2S - n_b} \\ S^+ = \sqrt{2S - n_b}\, b \\ S_z = S - n_b \end{cases}$$



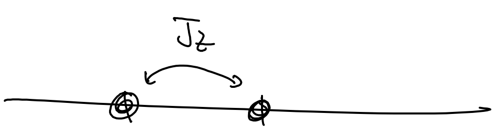
Spin 1/2

0 bosons ↑ 1 boson ↓

$S^- = b^+ \Leftarrow$ Simple

$$H = \frac{J}{2} \sum_{\langle ij \rangle} \left(b_i^+ b_j + h.c. \right) + J_z \sum_{\langle ij \rangle} \left(n_b^i - \tfrac{1}{2} \right)\left(n_b^j - \tfrac{1}{2} \right)$$

$+$ Constraint



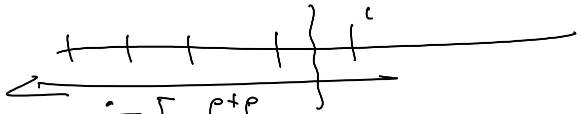
$$U \sum_i n_i (n_i - 1)$$

$U \to \infty$

Jordan-Wigner transformation. (⚠ $d = 1$)

$$\begin{cases} S_i^+ = f_i^+ \exp\left[i\pi \sum_{j<i} f_j^+ f_j \right] \\ S_i^- = f_i \;\; ----- \\ S_z = n_f - \tfrac{1}{2} \end{cases}$$

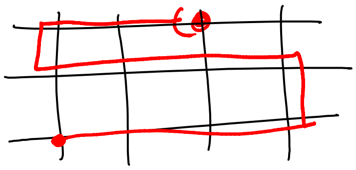
↑ ● ↓ —

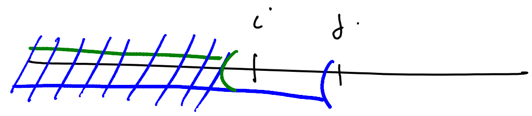


$$e^{i\pi \sum_{j<i} f_j^\dagger f_j}$$

String ensures that

$\alpha \neq \beta \qquad [S_\alpha, S_\beta] = 0$

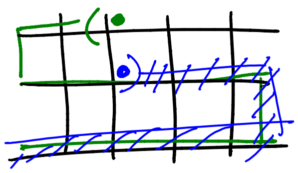




$$H = \frac{J}{2} \sum_{\langle ij \rangle} \underbrace{S_i^+ S_j^-}_{f_i^\dagger e^{i\pi \cdots} e^{i\pi \cdots} f_j} + h.c.$$

$$\frac{J}{2} \sum_{\langle ij \rangle} \left(f_i^\dagger f_j + h.c. \right) + J_z \sum_{\langle ij \rangle} (n_{if} - 1/2)(n_{jf} - 1/2)$$

$\hookrightarrow$ "Hubbard" model (t-V) model for fermions



$J_z = 0$

$$H = \frac{J}{2} \sum_{\langle ij \rangle} \left(S_i^+ S_j^- + S_i^- S_j^+ \right)$$

$$\downarrow$$

$$H = \frac{J}{2} \sum_{\langle ij \rangle} \left(f_i^\dagger f_j + f_j^\dagger f_i \right)$$

$\boxed{\text{fermions}} \longleftrightarrow \boxed{\text{Spins}} \xrightarrow[\text{(hard core)}]{} \boxed{\text{bosons}}$

Spin chain $\quad h = 0 \qquad \langle S_z \rangle = 0 \qquad \langle n_b - 1/2 \rangle = 0$

$n_b = 1/2 \qquad (n_f = 1/2)$