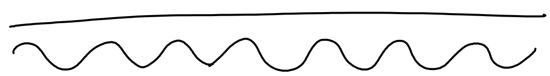
Beyond Luttinger Liquids.

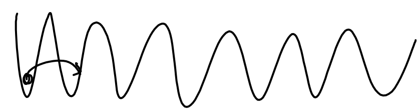
I] Effect of the lattice: Mott transition

$H_{kin} + H_{int}$



$$V(x) = V_0 \cos(Qx)$$

Lattice is big



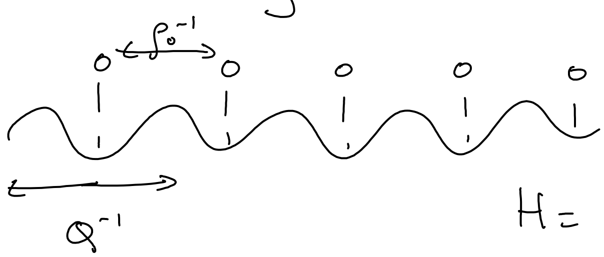
$$H = -t \sum_{\langle ij \rangle} \left(b_i^\dagger b_j + b_j^\dagger b_i \right) + U \sum_i n_i (n_i - 1)$$

$$\rho(x) = \rho_0 - \frac{1}{\pi} \nabla\phi + \rho_0 \sum_{p \neq 0} e^{ip(2\pi\rho_0 x - 2\phi(x))}$$

$$H_{lat} = \int dx \; V(x) \rho(x)$$

$$V_0 \cos(Qx) \quad \delta$$

$$\approx \int dx \; e^{i\overbrace{(Q - 2\pi\rho_0)}^{\delta} x - 2\phi(x)}$$

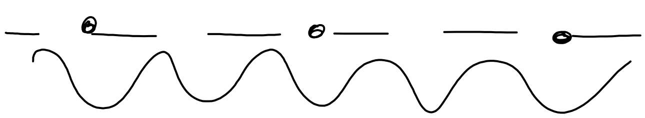


$\underline{\delta = 0}$ $\quad n = 1$, $n = 2$, $n = 3$

$$H = V_0 \int dx \; \cos(2\phi(x))$$

$2\pi\rho_0 \neq Q \qquad n \neq 1 \qquad n = 1 + \delta$

$$H = V_0 \int dx \; \cos(2\phi(x) + \delta x)$$



$$H = \int dx \cos(4\phi(x))$$

p order of commensurability with the lattice

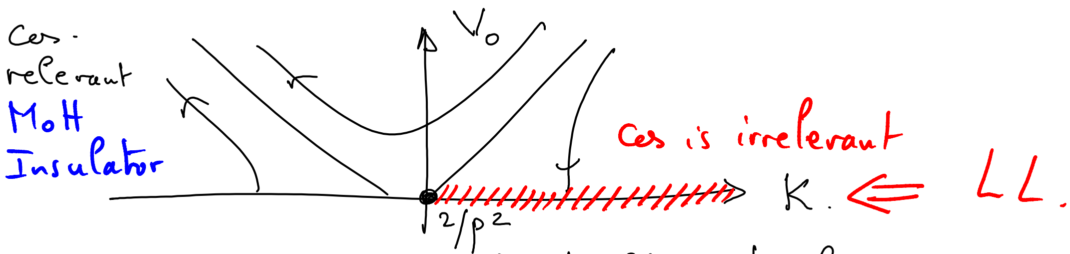
$$H = \int dx \; \cos(2p\,\phi(x))$$

1 boson every p sites

Commensurate Case.

$$H = \int uK\,\Pi^2(x) + \frac{u}{K}(\nabla\phi)^2 - V_0 \int dx \cos(2p\,\phi(x))$$

$$S = \frac{1}{2\pi K}\int dx\,d\tau\, \frac{1}{u}(\partial_\tau \phi)^2 + u(\nabla\phi)^2 - V_0\int d\tau \int dx \cos(2p\,\phi(x))$$



BKT Berezinskii - Kosterlitz - Thouless.

$$\langle \cos(2p\,\phi(x))\cos(2p\,\phi(0))\rangle \sim \left(\frac{1}{x}\right)^{2p^2K}$$

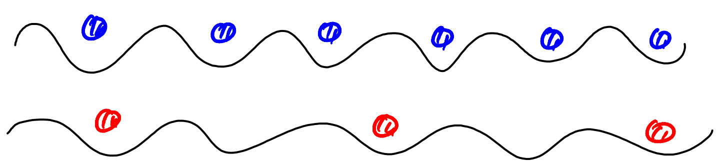
$$\cos(2p\,\phi(x)) \sim L^{-p^2K}.$$

$$\int d\tau\, dx \cos(2p\,\phi(x)) \sim L^{2-p^2K}.$$

$$S = \int [\omega^2 + k^2]\,\phi^*\phi + V_0\,\phi^2$$

$$\int [\omega_n^2 + k^2 + V_0]\,\phi^2$$

$$E^2 \simeq k^2 + V_0 \quad \Rightarrow \quad \text{Massive spectrum.}$$



$K < 2$

$K < 1/2$

↳ Book for details

Large Lattice

$$U\sum_i n_i(n_i - 1) \qquad \rho_0 - \frac{1}{\pi}(\nabla\phi) + \rho_0 e^{i(2\pi\rho_0 x - 2\phi)}$$

$$\int (\nabla\phi)(\nabla\phi)\,dx$$

$$U\sum_i \rho_0^2\, e^{i(\underbrace{2\pi\rho_0 x_i} - 2\phi(x_j))} + h.c. \qquad \rho_0 = \frac{1}{a}. \quad x_j = a\,j$$

$$U \sum_j \rho_0^2 \, e^{i(\underbrace{2\pi \rho_0 x_j}_{2\pi j} - 2\phi(x_j))} + h.c.$$

$$= U \rho_0^2 \sum_j e^{-i 2\phi(x_j)} + h.c. = U\rho_0^2 \int dx \, \cos(2\phi(x))$$

Spin Chains.

$$J_z \sum_{\langle ij \rangle} (S_z^i - 1/2)(S_z^j - 1/2)$$

$j = i+1$ ½

$$J_z \sum_i \left[-\frac{1}{\pi} \nabla\phi(r_i) + e^{\pm i 2\pi \rho_0 x_i - 2\phi(r_i)} \right] \left[-\frac{1}{\pi} \nabla\phi + e^{\pm 2\pi \rho_0 x_j - 2\phi(r_j)} \right]$$

$(-1)^i$ → oscillate.

$$\sum_i \underbrace{e^{i 2\pi \rho_0 x_i}}_{(-1)^i} \underbrace{e^{i 2\pi \rho_0 x_j}}_{(-1)^{i+1}} e^{-i 2\phi(x_i)} e^{-i 2\phi(x_j)}$$

$$- J_z \int dx \, \cos(4\phi(x))$$

$$H_{LL} - J_z \int dx \, \cos(4\phi(x)) \leftarrow L^{2-4K}.$$

$$\langle \cos(4\phi) \cos(4\phi) \rangle_{H_{LL}} = \left(\frac{1}{r}\right)^{8K}.$$

$\cos(4\phi)$ — L.L. — K.

relevant — 1/2

Ising (S_z) order.

↑ ↓ ↑ ↓ ↑ ↓ ↑

$$\begin{cases} \dfrac{\partial K}{\partial \ell} = -J_z^2 \\ \dfrac{\partial J_z}{\partial \ell} = J_z (2 - 4K) \end{cases}$$

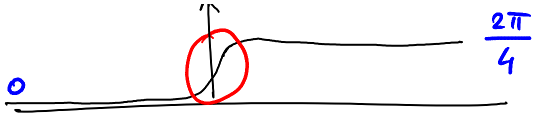
$$L = \alpha e^{\ell}$$

$$H_{LL} - \underset{K<2}{g_2 \cos(2\phi)} \; \underset{+}{-} \; \underset{K<1/2}{g_4 \cos(4\phi)}$$

$$S = \int dx\, d\tau \left[(\partial_\tau \phi)^2 + (\partial_x \phi)^2 \right] - g \cos(4\phi)$$

ϕ ~~~~~~~~ "phonons"

2π



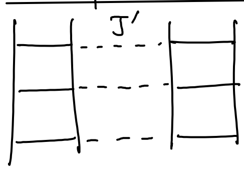
Kink (soliton)

$$S_t = -\frac{1}{\pi}\nabla\phi \qquad \Delta S_x = -\frac{1}{\pi}\int_{-\infty}^{+\infty}\nabla\phi = \frac{1}{\pi}\left[\phi(+\infty)-\phi(-\infty)\right]$$

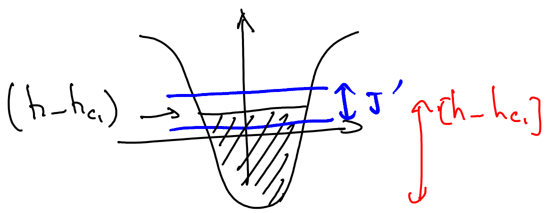
$\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow$

$$= \frac{1}{2}$$

II] Coupled chains.

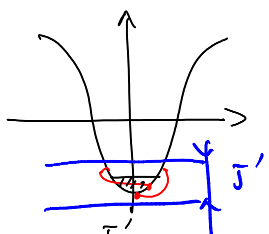


$$1/J' = \chi^{1D}(\beta_c)$$

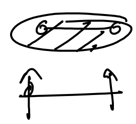


$J' \ll h-h_{c_1}$

$\Rightarrow$ Weakly coupled LL.

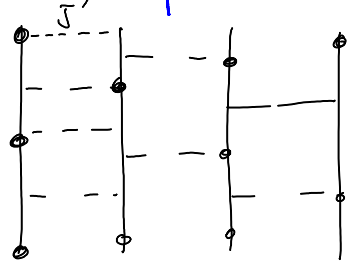


~~LL~~



Singlet

triplet

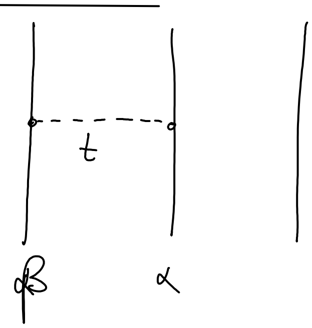


Anisotropic $\boxed{3D}$ System of hard core bosons.

$h-h_{c_1}$ small $n_b \searrow 0$

$\hookrightarrow$ "nearly free" bosons !! $\Rightarrow$ BEC !

Fermions :



$$H_{3D} - t\sum_{\langle\alpha\beta\rangle}\int dx\, \psi_\alpha^\dagger \psi_\beta.$$

$$H = \sum_\alpha H_{LL}^\alpha \leftarrow \int \Pi_\alpha^2 + (\nabla\phi_\alpha)^2$$

$$\psi_\alpha^\dagger = e^{i(\theta_\alpha - \phi_\alpha)} \qquad \psi_\beta = e^{-i(\theta_\beta - \phi_\beta)}$$

$$H_{3D} = -t \sum_{\alpha\beta} \cos(\theta_\alpha - \theta_\beta + \phi_\alpha - \phi_\beta)$$

Mott $\cos(\phi)$

$\phi \to \theta$ fluctuate

$\langle \psi_F \rangle = !$